E8 Cohomology and Physics

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Cohomology structure of E8 represents realistic E8 Lagrangian Physics
as described in http://vixra.org/abs/1602.0319

Weyl Symmetric Polynomial Degrees N of E8:
2, 8, 12, 14, 18, 20, 24, 30 and their product = I Weyl Group of E8 I
and their sum = I Weyl Reflections I + 8
(N-1 = 1, 7, 11, 13, 17, 19, 23, and 29 = exponents
are all relatively prime to E8 Coxeter Number = 30)

Topological Types (2N-1) of E8:
3, 15, 23, 27, 35, 39, 47, 59
center = Z1 = 1 = trivial

The cohomology structure of E8 describes the base manifold spacetime
and the gauge bosons and ghosts and the first-generation fermions
(second and third fermion generations are not fundamental, but are emergent)
of a realistic Lagrangian (see viXra 1602.0319 and 1701.0495 and 1701.0496).

E8 = 3 + 15 + 23 + 27 + 35 + 39 + 47 + 59 = 248

E8 has a maximal subalgebra D8 whose cohomology structure is

D8 3 7 11 15 15 19 23 27 = 120 = 28 + 28 + 64

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D8 represents Gauge Bosons+Ghosts and 8x8 Spacetime

D4 Gravity Gauge Bosons

	3	7	3	3	
and Standard Model Ghosts			4	8	

together make up a D4 subalgebra of D8:

D4grav	**3**	**7**	**7**	**11**	**= 28**

D4 Standard Model Gauge Bosons

	8	4			
and Gravity Ghosts	3	3	7	3	

together make up another D4 subalgebra of D8:

D4stdmod	**11**	**7**	**7**	**3**	**= 28**

8x8 Spacetime is represented by
D8 / D4 x D4 = 64 = 8dim momentum x 8dim position

4 4 4 12 16 24 = 64

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E8 / D8 represents 248 - 120 = 128 components for 8dim Spacetime of
8 first-generation fermion particles and 8 first-generation antiparticles:

Fermions 8 12 12 20 20 24 32 = 128

Here is more detail about E8 Cohomology structure of Fermions:

Fermions have 8 Spacetime components - Octonion Basis = 1 i j k E I J K

8F = 8 Fermion Types = e rUq gUq bUq n rDq gDq bDq
4Fe = 4 Electron Fermion Types = e rUq gUq bU
4Fn = 4 Neutrino Fermion Types = n rDq gDq bDq
Fp = Fermion particle
Fap = Fermion antiparticle

Fermions	8	12	12	20	20	24	32	= 128
1 x 8Fp	8							
ijk x 8Fp						24		
EIJK x 8Fp		12			20			
1ijk x 8Fap			12	20				
EIJK x 8Fap							32	

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Here is more detail about E8 Cohomology structure of 8x8 Spacetime = 8x8DST

The 8x8 is physically 8 momentum x 8 position of 8-dim Octonionic Spacetime.
The Octonion Basis elements represent 8 momentum components
of each M4 x CP2 position.

Octonion Basis = 1 i j k E I J K x M4 x CP2 Kaluza-Klein

8x8DST	4	4	4	12	16	24	= 64
1 x M4	4						
1 x CP2		4					
E x CP2			4				
ijk x M4				12			
EIJK x M4					16		
ijkIJK x CP2						24	

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References:
Mimura and Toda, “Topology of Lie Groups, I and II”, AMS 1991
Humphreys, “Reflection Groups and Coxeter Groups”, Cambridge 1990
Kane, “The Homology of Hopf Spaces”, North-Hoilland 1988

Topology of Lie Groups, I and II by Mamoru Mimura and Hirosi Toda (AMS 1991)

Mod 0 **decomposition of compact Lie groups.** The following is the classical Hopf theorem from which originated the theory of H-spaces and Hopf algebras.

THEOREM 6.26 (Hopf). *Let* $\mathbf{k}$ *be a field of characteristic* 0. *For a compact connected Lie group* G, *there exist elements* $x_{2n_i+1} \in H^{2n_i+1}(G; \mathbf{k})$ *of odd degree for* $i = 1, \ldots, l$, *such that*

$$H^*(G; \mathbf{k}) = \Lambda(x_{2n_1+1}, \ldots, x_{2n_l+1}).$$

The results also hold for an arcwise connected H*-space* G *with* $\dim_{\mathbf{k}} H^*(G; \mathbf{k}) < \infty$. *Moreover, one can replace* $H^*(G; \mathbf{k})$ *by a connected, commutative Hopf algebra* A^* *of finite dimension over* $\mathbf{k}$.

DEFINITION. A sequence $(2n_1 + 1, \ldots, 2n_l + 1)$ of odd degrees, in the theorem above, written in the order of the size is called the *type* of G and the number of the generators the *rank* of G, denoted by $l = \operatorname{rank} G$.

THEOREM 6.27. *Let* G *be a* 1*-connected Lie group of type* $(2n_1 + 1, \ldots, 2n_l + 1)$. *Then there exists a* 0*-equivalence*

$$f : S^{2n_1+1} \times \cdots \times S^{2n_l+1} \simeq_0 G$$

that is, G *is* 0*-regular.*

The Homology of Hopf Spaces by Richard M. Kane (North-Holland 1988)

APPENDIX A: MOD P COHOMOLOGY OF LIE GROUPS

The Cartan-Killing classification of locally isomorphic compact Lie groups gives 7 classes of simple Lie groups: A_n $(n \geq 1)$, B_n $(n \geq 2)$, C_n $(n \geq 3)$, D_n $(n \geq 4)$, G_2, F_4, E_6, E_7 and E_8. Each class contains a simply connected representative unique up to global isomorphism. The other groups in the class are quotients of the former by subgroups of its centre.

Group	Type	Centre
$A_n = SU(n+1)$	$(3,5,7,\ldots,2n+1)$	$\mathbb{Z}/n+1$
$B_n = Spin(2n+1)$	$(3,7,11,\ldots,4n-1)$	$\mathbb{Z}/2$
$C_n = Sp(n)$	$(3,7,11,\ldots,4n-1)$	$\mathbb{Z}/2$
$D_n = Spin(2n)$	$(3,7,11,\ldots 4n-5,2n-1)$	$\mathbb{Z}/2 \oplus \mathbb{Z}/2$ n even
		$\mathbb{Z}/4$ n odd
G_2	$(3,11)$	0
F_4	$(3,11,15,23)$	0
E_6	$(3,9,11,15,17,23)$	$\mathbb{Z}/3$
E_7	$(3,11,15,19,23,27,35)$	$\mathbb{Z}/2$
E_8	$(3,15,23,27,35,39,47,59)$	0

<u>The Exceptional Groups</u> (Kono [3], Mimura-Toda [1], Thomas [3])

First of all we have

$$H^*(E_8;\mathbb{F}_p) = E(x_3,x_{15},x_{23},x_{27},x_{35},x_{39},x_{47},x_{59}) \qquad \text{for } p \geq 7$$

$$H^*(E_8;\mathbb{F}_2) = E(x_{17},x_{23},x_{27},x_{29}) \otimes \mathbb{F}_2[x_3,x_5,x_9,x_{15}/_{(x_3{}^{16},x_5{}^{8},x_9{}^{4},x_{15}{}^{4})}$$

$$H^*(E_8;\mathbb{F}_3) = E(x_3,x_7,x_{15},x_{19},x_{27},x_{35},x_{39},x_{47}) \otimes \mathbb{F}_3[x_8,x_{20}]/_{(x_8{}^3,x_{20}{}^3)}$$

$$H^*(E_8;\mathbb{F}_5) = E(x_3,x_{11},x_{15},x_{23},x_{27},x_{35},x_{39},x_{47}) \otimes \mathbb{F}_5[x_{12}]/_{(x_{12}{}^5)}$$

E8 topological structure - Fermions shown in Red - Spacetime shown in Blue)

3 - S3 = SU(2) inside U(2,2) inside D4 subalgebra of D8 subalgebra of E8
That D4 is D4grav and it represents
16-dim U(2,2) Conformal Group gauge bosons of Gravity + Dark Energy
and 12 Standard Model Ghosts

15 - S15 fibration gives S7 and S8
S7 is inside U(2,2) of D4grav
S8 is 8 time-components of 8 Fermion Particles

23 - S23 fibration gives S11 and S12
S11 fibration gives S3 and S8
S3 is inside U(2,2) of D4grav
S8 gives 4 and 4
S4 is 4 Standard Model Ghosts of D4grav
S4 is 1 x M4 Spacetime components
S12 is 12 of the 32 CP2 components of 8 Fermion Particles

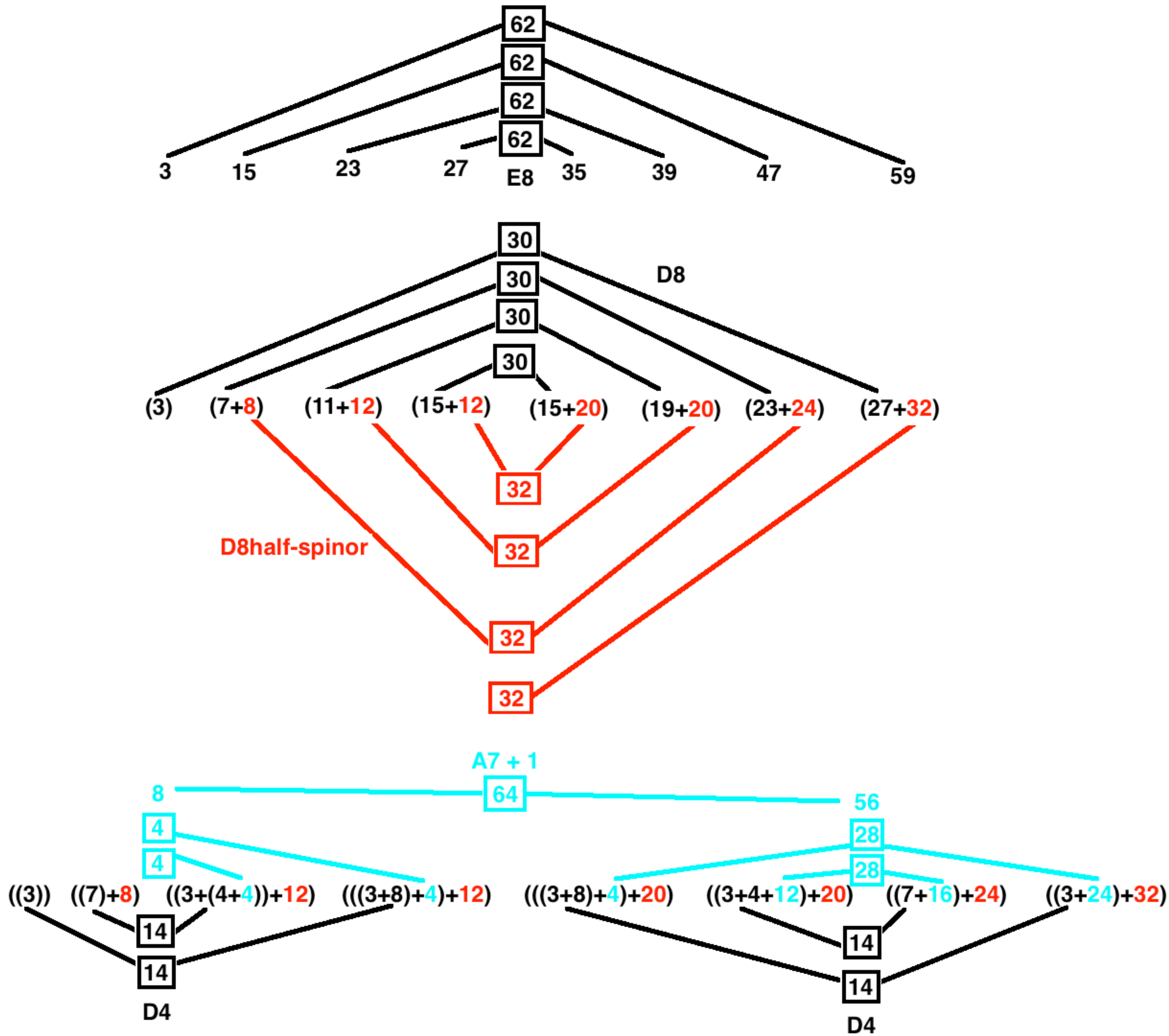
27 - S27 fibration gives S15 and S12
S15 fibration gives S7 and S8
S7 fibration gives S3 and S4
S3 is inside U(2,2) of D4grav
S4 is 1 x CP2 Spacetime components
S8 is 8 Standard Model Ghosts of D4grav
S12 is 12 of the 32 M4 components of 8 Fermion AntiParticles

35 - S35 fibration gives S15 and S20
S15 fibration gives S7 and S8
S7 fibration gives S3 and S4
S3 is 3 Gravity Ghosts of D4stdmod
S4 is E x CP2 Spacetime components
S8 is 8 Standard Model Gauge Bosons of D4stdmod
S20 is 20 of the 32 M4 components of 8 Fermion AntiParticles

39 - S39 fibration gives S19 and S20
S19 gives S7 and S12
S7 fibration gives S3 and S4
S3 is 3 Gravity Ghosts of D4stdmod
S4 is 4 Standard Model Gauge Bosons of D4stdmod
S12 is ijk x M4 Spacetime components
S20 is 20 of the 32 CP2 components of 8 Fermion Particles

47 - S47 fibration gives S23 and S24
S23 gives S7 and S16
S7 is 7 Gravity Ghosts of D4stdmod
S16 is EIJK x M4 Spacetime components
S24 is 24 space components of 8 Fermion Particles

59 - S59 fibration gives S27 and S32
S27 gives S3 and S24
S3 is 3 Gravity Ghosts of D4stdmod
S24 is ijkIJK x CP2 Spacetime components
S32 is 32 CP2 components of 8 Fermion AntiParticles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E8
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D8
(3)
(7+8)
(11+12)
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D8half-spinor
A7 + 1
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64
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((3+4+12)+20)
((7+16)+24)
((3+24)+32)
14
14
D4
14
14
D4