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$10 = 6 + 4$

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Abstract

Some physics models have 10 dimensions that are usually decomposed into:
4 spacetime dimensions with local Lorentz $Spin(1,3)$ symmetry
plus
a 6-dimensional compact space related to internal symmetries.

A possibly useful alternative decomposition is into:
6 spacetime dimensions with local Conformal symmetry
of the Conformal Group $C(1,3) = Spin(2,4) = SU(2,2)$
plus
a 4-dimensional compact Internal Symmetry Space
that can be taken to be complex projective 2-space $\mathbf{C}P^2$
which, since $\mathbf{C}P^2 = SU(3)/U(2)$,
is a natural representation space for $SU(3)$
and on which $U(2) = SU(2) \times U(1)$ can be represented naturally by local action.

Contents

1 Decomposition of 10 Dimensions

Some physics models have 10 dimensions that are usually decomposed into:

4 spacetime dimensions with local Lorentz $Spin(1,3)$ symmetry

plus

a 6-dimensional compact space related to internal symmetries.

A possibly useful alternative decomposition is into:

6 spacetime dimensions with local $C(1,3) = Spin(2,4) = SU(2,2)$ Conformal symmetry.

plus

a 4-dimensional compact Internal Symmetry Space.

1.1 6-Dimensional Conformal spacetime

Conformal symmetries and some of their physical applications are described in the book of Barut and Raczka [1].

The Conformal group $C(1,3)$ of Minkowski spacetime is the group $SU(2,2) = Spin(2,4)$. As $Spin(2,4)$, the Conformal group acts on a 6-dimensional (2,4)-space that is related to the 6-dimensional $\mathbf{C}P^3$ space of Penrose twistors [2].

It is reasonable to consider the 6-dimensional Conformal space as the spacetime in the dimensional decomposition of 10-dimensional models because Conformal symmetry is consistent with such physics structures as:

Maxwell's equations of electromagnetism;

the quantum theoretical hydrogen atom;

the canonical Dirac Lagrangian for massive fermions, as shown by Liu, Ma, and Hou [3];

gravity derived from the Conformal group using the MacDowell-Mansouri mechanism, as described by Mohapatra [4];

the Lie Sphere geometry of spacetime correlations; and

the Conformal physics model of I. E. Segal [5].

1.2 4-Dimensional Internal Symmetry Space

An example of a possibly useful 4-dimensional compact Internal Symmetry Space is complex projective 2-space $\mathbf{C}P^2$.

Since $\mathbf{C}P^2 = SU(3)/U(2)$, it is a natural representation space for $SU(3)$.

Further, $U(2) = SU(2) \times U(1)$ can be represented naturally on $\mathbf{C}P^2 = SU(3)/U(2)$ as a local action.

Therefore, all three of the gauge groups of the Standard Model $SU(3) \times SU(2) \times U(1)$ can be represented on the 4-dimensional compact Internal Symmetry Space $\mathbf{C}P^2 = SU(3)/U(2)$.

The following section lists some examples of physics models that have such 10-dimensional spaces: Superstring theory; the Division Algebra model of Geoffrey Dixon; and the $D_4 - D_5 - E_6 - E_7$ physics model.

2 Superstrings, Dixon, and D4-D5-E6-E7

2.1 Superstrings

The 10-dimensional space of Superstring theory is well known, and described in many references, so I will not try to summarize it here. One particularly current and thorough reference is the 2-volume work of Polchinski [6].

2.2 Geoffrey Dixon's Division Algebra model

Geoffrey Dixon, in his publications and website [7], considers the real division algebras:
the real numbers $\mathbf{R}$;
the complex numbers $\mathbf{C}$;
the quaternions $\mathbf{Q}$; and
the octonions $\mathbf{O}$.

Dixon then forms the tensor product $\mathbf{T} = \mathbf{R} \otimes \mathbf{C} \otimes \mathbf{Q} \otimes \mathbf{O}$ and considers the 64-real-dimensional space $\mathbf{T}$.

Then Dixon takes the left-adjoint actions $\mathbf{T_L} = \mathbf{C_L} \otimes \mathbf{Q_L} \otimes \mathbf{O_L}$, and notes that $\mathbf{T_L}$ is isomorphic to $\mathbf{C}(16) = Cl(0,9) = \mathbf{C} \otimes Cl(0,8)$.

Then Dixon considers the algebra $\mathbf{T}$ to be the spinor space of $\mathbf{T_L}$.

Then Dixon forms a matrix algebra $\mathbf{T}_L(2)$ as the 2×2 matrices whose elements are in the left-action adjoint matrix algebra $\mathbf{T_L}$ and notes that $\mathbf{T_L}(\mathbf{2})$ is isomorphic to $\mathbf{C}(32) = \mathbf{C} \otimes Cl(1,9)$.

Dixon describes the matrices $\mathbf{T_L}(\mathbf{2})$ as having spinor space $\mathbf{T} \oplus \mathbf{T}$ and $\mathbf{C} \otimes Cl(1,9)$ as the Dirac algebra of 10-dimensional (1,9)-space.

Dixon then describes leptons and quarks in terms of reduction of the Dirac spinors of the 10-dimensional (1,9)-space to the Dirac spinors of a

4-dimensional (1,3)-spacetime.

The right-action adjoint matrix algebra $\mathbf{T_R}$ is not the same as the left-action adjoint $\mathbf{T_L}$, because, although $\mathbf{C_R} = \mathbf{C_L}$ and $\mathbf{O_R} = \mathbf{O_L}$, it is a fact that $\mathbf{Q_R} \neq \mathbf{Q_L}$ (they are isomorphic but not identical).

Since $\mathbf{Q_R} = \mathbf{Q}$, the part of the matrix algebra $\mathbf{T_R}$ that differs from $\mathbf{T_L}$ is just $\mathbf{Q}$, and the different part of the 2×2 matrix algebra $\mathbf{T}_R(2)$ is just the 2×2 matrix algebra with quaternion entries $\mathbf{Q}(2)$.

In section 6.7 of his book [7], Dixon shows that commutator closure of the set of traceless 2×2 matrices over the quaternions $\mathbf{Q}$, which he denotes by $sl(2, \mathbf{Q})$, is the Lie algebra of $Spin(1,5)$.

Since the Lie algebra $Spin(1,5)$ is just the Lie algebra of the Conformal group $C(1,3) = Spin(2,4) = SU(2,2)$ with a different signature,

I conjecture that it might be useful to consider the spacetime part of Dixon's 10-dimensional (1,9)-space to be the 6-dimensional (1,5)-spacetime of $Spin(1,5)$.

That would leave a 4-dimensional (0,4)-space to be used as an Internal Symmetry Space.

2.3 the D4-D5-E6-E7 model

The D_5 Lie algebra of the $D_4 - D_5 - E_6 - E_7$ physics model corresponds (with Conformal signature) to the Lie algebra $Spin(2,8)$ of the Clifford algebra $Cl(2,8)$ whose vector space is 10-dimensional.

As the $D_4 - D_5 - E_6 - E_7$ physics model is described on the web [8], I will not try to summarize it here.

3 Acknowledgements

The idea of 6-dimenisonal spacetime with Conformal symmetry was motivated by the works of I. E. Segal [5] and by e-mail conversations with Robert Neil Boyd.

The idea of 4-dimensional Internal Symmetry Space was motivated by Cayley calibrations of octonions [9] and by e-mail conversations with Matti Pitkanen.

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