High Energy Particle Physics

Unimodular SL(n,R) Gravity and E8 Physics

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Unimodular SL(n,R) Gravity in n-dimensional SpaceTime is related to E8 Physics in two ways: to 8-dim SpaceTime and to 4-dim Physical Minkowski M4 SpaceTime. The Unimodular SL(8,R) represents 8-dim SpaceTime as a generalized checkerboard of SpaceTime HyperVolume Elements. The Unimodular SL(4,R) has conformal structure effectively equivalent to that of MacDowell-Mansouri SU(2,2) Conformal Gravity and, further, is useful in resolving the strong CP problem.

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in two ways: to 8-dim SpaceTime and to 4-dim Physical Minkowski M4 SpaceTime.

With respect to SL(8,R) and 8-dim SpaceTime:
248-dim E8 = 120-dim D8 bivectors of Cl(16) + 128-dim half-spinors of Cl(16)
D8 contains two copies of 28-dim D4 and D8 / D4xD4 = (1+63)-dim R + SL(8,R)
The SL(8,R) represents Unimodular SL(8,R) Gravity of 8-dim SpaceTime.
Anderson and Finkelstein in Am . J. Phys. 39 (1971) 901-904 said: "... Unimodular relativity ...
expresses the existence of a fundamental element of spacetime hypervolume
at every point. ..."
Therefore SL(8,R) effectively describes the 8-dim SpaceTime of E8 Physics
as a generalized checkerboard of SpaceTime HyperVolume Elements,
while the two copies of D4 describe Gauge Bosons and
(64+64)-dim half-spinors describe 8 components of 8 Fundamental Fermion Particles
and 8 components of 8 fundamental fermion antiparticles.
Equivalently,
you can see 64-dim R+SL(8,R) in terms of factoring Cl(16)
by Real Clifford Algebra 8-Periodicity into the tensor product Cl(8) x Cl(8) = Cl(16)
as the tensor product of the 8-dim vector spaces 8v of each of the Cl(8) factors
so that 64-dim R+SL(8,R) = 8v x 8v and if you regard the two Cl(8) as Fourier duals
then 8v describes 8-dim Spacetime Position and the other 8v describes its Momentum.

SL(8,R) also appears in E8 Maximal Contraction = semi-direct product H92 x SL(8,R)
where H92 is (8+28+56 +1+ 56+28+8)-dim Heisenberg Creation/Annihilation Algebra
so that H92 x SL(8,R) has 7-graded structure:

grade -3 = Creation of 1 fermion (tree-level massless neutrino)
with 8 SpaceTime Components
for a total of 8 fermion component creators (related to SpaceTime by Triality)
grade -2 = Creation of 8+3+1 = 12 Palev Bosons for Standard Model
and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity
for a total of 28 Palev Bosons
grade -1 = Creation of 7 massive Dirac fermions each with 8 SpaceTime Components
for a total of 56 fermion component creators
grade 0 = 1 + SL(8) = 1+63 = 64-dim representing 8-dim SpaceTime of HyperVolume Elements
grade 1 = Annihilation of 7 massive Dirac fermions each with 8 SpaceTime Components
for a total of 56 fermion component creators
grade 2 = Annihilation of 8+3+1 = 12 Palev Bosons for Standard Model
and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity
for a total of 28 Palev Bosons
grade 3 = Annilation of 1 fermion (tree-level massless neutrino)
with 8 SpaceTime Components
for a total of 8 fermion component creators (related to SpaceTime by Triality)

With respect to SL(4,R) and 4-dim Physical Minkowski M4 SpaceTime:
At low energies (where we do experiments, far below Planck energy)
the SpaceTime of E8 Physics is (4+4)-dim M4 x CP2 Kaluza-Klein
and
the two copies of D4 that live in the D8 subalgebra of E8 describe
the Standard Model Gauge Bosons (by one D4, here called D4sm)
and
Gauge Bosons for MacDowell-Mansouri Gravity (by the other D4, here called D4g).

D4g contains as subalgebra D3 = A3 which, depending on signature, can be either

Conformal SU(2,2) = Spin(2,4) of Cl(2,4) = M(4,Q) = 4x4 Quaternionic Matrices
which gives Conformal Gravity for 4-dim M4 SpaceTime by MacDowell-Mansouri
or
Unimodular SL(4,R) = Spin(3,3) of Cl(3,3) = M(8,R) = 8x8 Real Matrices
which gives Unimodular SL(4,R) Gravity for 4-dim M4 SpaceTime

For 4-dim M4 SpaceTime,
Conformal SU(2,2) = Spin(2,4) Gravity and Unimodular SL(4,R) = Spin(3,3) Gravity
seem to be effectively equivalent since, as Bradonjic and Stachel in arXiv 1110.2159 said:
"... in ... **Unimodular relativity** ...
the symmetry group of space-time is ... the special linear group **SL(4,R)** ...
the metric tensor ... break[s up] ... into
the **conformal structure** represented by a conformal metric ... with det = -1
and **a four-volume element** ... at each point of space-time ...[that]... may be
the remnant, in the ... continuum limit,
of a more fundamental discrete quantum structure of space-time itself ...".

The four-volume element of Unimodular SL(4,R) Gravity corresponds
to the 10-dim Anti-deSitter Spin(2,3) subalgebra Conformal Spin(2,4) Gravity
and
the conformal structure of Unimodular SL(4,R) Gravity corresponds
to the Special Conformal Dark Energy part of Conformal Spin(2,4) Gravity.

The Unimodular SL(4,R) point of view is useful in understanding Strong CP.
Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said:
"... Because of the existence of topologically nontrivial solutions, instantons, of the
classical field equations associated with quantum chromodynamics (QCD), the
quantized theory contains a dimensionless parameter ø ($0 \leq ø < 2\pi$) not explicit in the
classical lagrangian. Since ø multiplies an expression odd in CP, QCD predicts violation
of that symmetry unless the phase ø takes one of the special values ... 0 (mod π) ...
this fine tuning is the strong CP problem ...
the quantum dynamics of ... unimodular gravity ... may lead to the relaxation
of ø to ø = 0 (mod π) without the need either for a new particle ... such as the axion ...
or for any appeal to wormholes ...".