

McKay Correspondence between Physical World and Mental World

Frank Dodd (Tony) Smith, Jr. - around Halloween 2010

Saul-Paul Sirag, in an Appendix to Jeffrey Mishlove's book "Roots of Consciousness" (Council Oak Books 1993), describes the relationship between the Physical World and the Mental World in terms of the correspondence between McKay Subgroups of $SU(2)$ and Lie Algebras using the complex 133-dimensional exceptional Lie algebra E_7 as fundamental. This paper is an attempt to follow Saul-Paul Sirag's approach except for one difference: using real 248-dimensional exceptional Lie algebra E_8 as fundamental. The basic idea is set out by Saul-Paul Sirag in the Appendix:
"...[this quote is modified by using E_8 terminology]...
There is a Universal Body [120-dim ID group with 8-dim catastrophe perturbation space]...
There is a Universal Mind [Lie algebra E_8 generated by reflections in R_8]...
The intersection of the two
...[8-dim R_8 as ID Catastrophe Control Perturbation Space and
8-dim R_8 as E_8 reflection space and]...
is Universal Consciousness
...
There is a Universal Geometric Entity ...[UGE]... which is the direct product of ...
the Universal Body ... and ... the Universal Mind
...
Notice that from the point of view of ...[UGE]... the system is monistic,
but from the point of view of the constituent algebras ...[ID and E_8]... the system
is dualistic. ...".

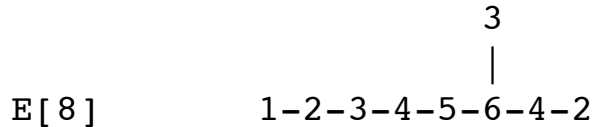
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E8 Lie Algebra Balance Numbers and Weyl Group

John McKay said on usenet sci.math in 1993:

"... For each finite subgroup of SU2, we get an affine Dynkin diagram ...



...

The [McKay] correspondence is ...

E[8] ...[corresponds to] 2.Alt[5] = SL(2,5) binary icosahedral [ID group]

...

There are [8+1 = 9 **balance numbers** for E8]...

The sum of the numbers [1+2+3+4+5+6+4+2+3 = 30 is] h = Coxeter number.

The sum of the squares is the order of ...[120-element ID for E8]...

The numbers are the degrees of irreducible representations of G.

They are first Chern numbers (singularities).

They are the periods of products of pairs

of Fischer involutions mod centre ... E[8] ...[for]... Monster

...

for the E8 - icosahedral ... case, the singularity is $x^2+y^3+z^5=0$...".

In a 1993 email John McKay said:

"... The connection with the monster is not understood at all, however it has as consequence that each node on the extended diagrams is attached to a modular function ...".]

Saul-Paul Sirag, in the Appendix,

noted that the number of mirrors in E8 reflection space is $8h/2 = 120$.

He also noted that the product of the E8 balance numbers is $128 \times 27 \times 5$

which when multiplied by $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ gives

the order 696,729,600 of the Weyl reflection group of E8 in R8 Euclidean 8-dimensional space.

Saul-Paul Sirag, in the Appendix, used a correspondence between E7 balance numbers and degrees of catastrophe terms that is not a general correspondence and in particular breaks down for E8, as Peter Slodowy said in a 1993 email message to John McKay.

Since that is not a valid correspondence in general and not valid for E8 in particular, that correspondence will not be discussed further in this paper.

Icosahedral Double Group Catastrophe Control Space

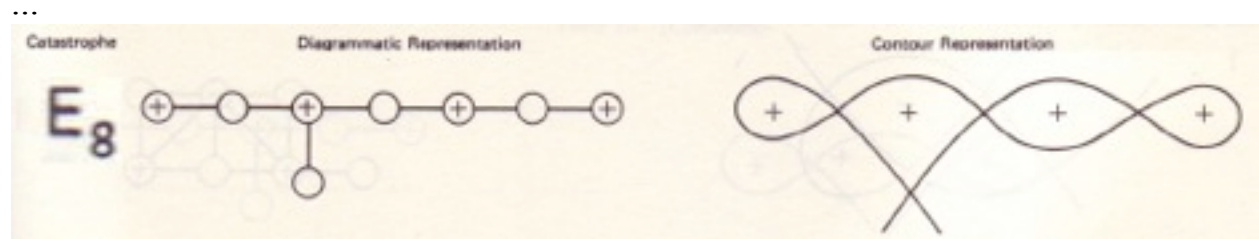
Robert Gilmore, in his book "Catastrophe Theory" (Dover 1981) said:

"...[The Icosahedral Double Group Catastrophe]... E8 ...[has]...

Catastrophe Germ ... $X^3 + Y^5$

...[with]... Perturbation ...

$a_1 Y + a_2 Y^2 + a_3 Y^3 + a_4 X + a_5 X Y + a_6 X Y^2 + a_7 X Y^3$



The germs E_6, E_8 are

$$E_6: f(x, y) = x^3 + y^4$$

$$E_8: f(x, y) = x^3 + y^5$$

The rules for determinacy and unfolding are particularly easy to carry out for E_6 and E_8 because both $\partial f / \partial x$ and $\partial f / \partial y$ are monomials. These calculations are summarized diagrammatically in Fig. 23.4.

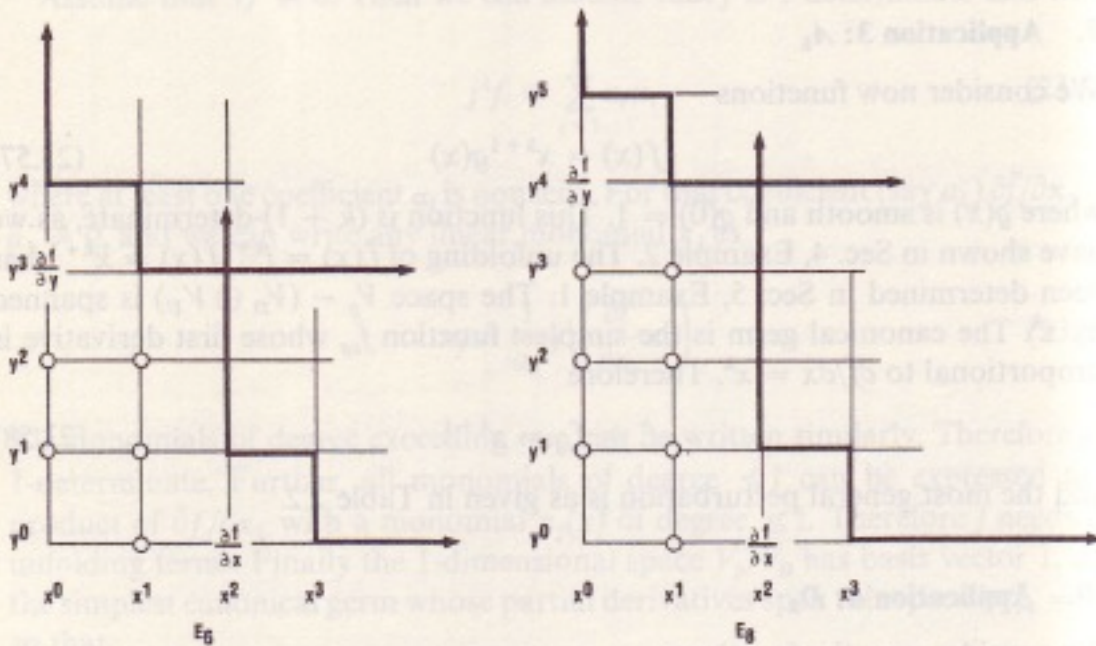


Figure 23.4 For E_6 and E_8 all monomials of degree 4 and 5 can be expressed in the form $(\partial f / \partial x_i) m_j$. The unfolding terms are represented by open circles. We exclude the constant term.

...

for E8 ...[with]... control parameter space R7 ...[basis {a1,a2,a3,a4,a5,a6,a7}]...
the maximum number ... of isolated critical points ..[is]... 8 ...”.

In the Appendix, Saul-Paul Sirag did not “exclude the constant term” as Robert Gilmore did, so, if we follow Saul-Paul Sirag’s approach and effectively add a control parameter a0, we see that the ID E8 Catastrophe Control Parameter Space is R8 with basis {a0,a1,a2,a3,a4,a5,a6,a7}

and

the Catastrophe Control Parameter Polynomials of ID are invariant under the Weyl Group of the E8 Lie Algebra Root Vector Reflections.

Since the E8 Weyl Group generates the E8 Lie Algebra from the Root Vector Reflections in R8,

and

also generates the Catastrophe Control Parameter Polynomials of ID

it is clear that R8 Reflections are the common ground from which the ID Universal Body and the E8 Universal Mind both emerge,

so

it is natural that

the R8 Reflection Space be identified as the Universal Consciousness.

R8 Universal Consciousness and World Line Strings

R8 Reflection Space of Universal Consciousness combines
Root Vector Reflections of the E8 Universal Mind
with
Weyl Group Catastrophe Control Polynomials of the ID Universal Body.

A way to see this interaction in greater detail is to expand R8 to 10 dimensions and then formulate a Bohmian Quantum Potential in terms of a 26-dimensional Bosonic String Theory that gives a superposition/separation (see <http://tony5m17h.net/Rzeta.html#supsepBSgrav>) gravity-type potential needed for Penrose/Hameroff Quantum Consciousness.

In terms of the E8 sector, the expansion from 8 to 10 dimensions starts with 8-dim as sum of 4-dim Minkowski plus 4-dim CP2 Internal Symmetry and then expands the 4-dim Minkowski to 6-dim Conformal Spin(2,4) spacetime resulting in $6+4 = 10$ dimensions.

In terms of the Icosahedral Double sector, the expansion from 8 to 10 dimensions starts with Catastrophe Control Parameter space (basis $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$) and adds two basis elements $\{X, Y\}$ of ID Catastrophe Germ space whose polynomials are invariant under the Icosahedral Double Group ID resulting in $8+2 = 10$ dimensions.

The strings in this version of Bosonic String Theory correspond to Possible World-Lines in the Many-Worlds and there is no naive 1-1 supersymmetry. (see <http://tony5m17h.net/E6StringBraneStdModelAR.pdf>)

ID Universal Body and Standard Model unified with Gravity

We can construct a realistic physics model,
including the Standard Model unified with Gravity,
from the 600cell with 120 vertices,
which 120 vertices correspond to the 120 elements of ID.

(see <http://tony5m17h.net/TetraJJDECF.pdf>)

E8 Universal Mind and Clifford Algebra

E8 is part of the Cl(16) Clifford algebra (bivector plus halfspinor). Clifford algebras describe quantum computers which in turn describe consciousness. Any Clifford algebra no matter how large can be described in terms of Cl(16) factors due to Cl(8) periodicity of real Clifford algebras.

For Quantum Consciousness details see
<http://tony5m17h.net/AfricaCl8x8E8.pdf>
<http://tony5m17h.net/E8CCTS12a.pdf>
<http://www.valdostamuseum.org/hamsmith/QuanCon.html>
<http://tony5m17h.net/QuantumMind2003.html>

Here is a rough outline of Clifford Algebra Quantum Consciousness:

The human brain contains about 10^{18} tubulins in cylindrical microtubules. Each tubulin contains a Dimer that can be in one of two binary states.



The Microtubule

in the illustration (from a Rhett Savage web page), the red dimer has its electron in the down state and the blue dimer has its electron in the up state. Each tubulin is about $8 \times 4 \times 4$ nanometers in size and contains about 450 molecules (amino acids) each with about 20 atoms.

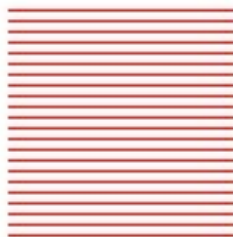
If about 10% of the brain is involved in a given conscious thought, then, since $2^{56} = 7.2 \times 10^{16}$, it involves about 10% of 10^{18} or about $2^{56} = 2^{(8 \times 7)}$ tubulins, so the mathematics of that thought is described by the Clifford algebra $Cl(56)$ which is (by 8-periodicity) $Cl(56) = Cl(8 \times 7) = Cl(8) \times \dots (7 \text{ times tensor product}) \dots \times Cl(8) = 7$ states of the basic Clifford algebra $Cl(8)$

That may account for "The Magical Number Seven, Plus or Minus Two: Some Limits on our Capacity for Processing Information" by George Miller available on the web at <http://psychclassics.yorku.ca/Miller/> where he refers to a

“... basic process that limits our unidimensional judgements to about seven categories ...”.

He distinguishes “the span of absolute judgment” with limit of about seven from “the span of immediate memory” as to which Wikipedia says the limit “... is probably around three or four”.

Since $2^{(8 \times 4)} = 2^{32} = 4.3 \times 10^9$ and $2^{(8 \times 3)} = 2^{24} = 1.7 \times 10^7$ it may be that the span of immediate memory corresponds to the square root of the number of tubulins $\sqrt{10^{18}} = 10^9$ because if the tubulins required for a full judgment thought process were to be represented as an area



then the tubulins required for immediate memory (lasting only a short time and allowing most (almost all) brain capacity to be used for other things) might be represented by a line (square root of the area)



As to how the thought formation process works, my model (which is of the Penrose-Hameroff type) indicates that all 2^{56} of the tubulins are coherently in phase together, forming a coherent quantum state containing all possible thought outcomes (i.e., all Bohm possibilities or all possible Worlds of the Many-Worlds).

After a time the coherent state decoheres into a single outcome state that is the thought that is the result of the process (Penrose calls that Orch OR, or "Orchestrated Objective Reduction of Quantum Coherence".)

Penrose proposes that Orch OR is caused by Quantum Gravity, occurring after expiration of the time allowed for that many tubulin states to be held in a coherent superposition.

The time at which decoherence takes place and a thought is formed can be calculated using quantum gravity ideas, and the calculation results are consistent with the data of the human brain. For details see <http://tony5m17h.net/QuanCon.html>

The formation of a human thought by 2^{56} tubulins is similar to the formation of our Universe beginning with an initial single Quantum Fluctuation Big Bang corresponding to the Clifford Algebra $Cl(0)$ with $2^0 = 1$ element, then non-unitary Octonionic inflation proceeding to

$Cl(1)$ with $2^1 = 2$ elements

$Cl(2)$ with $2^2 = 4$ elements

$Cl(4)$ with $2^4 = 16$ elements

$Cl(8)$ with $2^8 = 256$ elements

$Cl(16) = Cl(8) \times Cl(8)$ with $2^{16} = 65,536$ elements

$Cl(32) = Cl(8) \times \dots (4 \text{ times tensor product}) \dots \times Cl(8)$ with $2^{32} = 4.3 \times 10^9$

$Cl(64) = Cl(8) \times \dots (8 \text{ times tensor product}) \dots \times Cl(8)$ with $2^{64} = 1.8 \times 10^{19}$

at which level octonion self-reflexivity of $Cl(64) = Cl(8 \times 8)$ causes decoherence and the beginning of $M_4 \times CP_2$ Kaluza-Klein quaternionic unitary expansion and the end of inflation as described by Paula Zizzi (see http://en.wikipedia.org/wiki/Paola_Zizzi and <http://www.valdostamuseum.org/hamsmith/cosm.html#instexp>).

The model is consistent with the Big Bang and Inflation being a Thought in the Mind of God

Paula Zizzi wrote a paper "Poetry of a Logical Truth" in which she said "... We believe that the sense of the "Self" in human minds arises when a quantum mental state undergoes a mirror measurement.

...

"mirror measurement" would correspond to a "mirroring" of the "Self", and some anomaly in that process might be responsible for a mental disease like the "borderline".

...

The communication channel between a mind in a classical state and another mind in a quantum state, corresponding to a (external) quantum measurement, in the framework of standard quantum logic, can be viewed as the old kind of therapy.

... A new kind of therapy might require the therapist to reconstruct the paraconsistent logic of the patient from the data ...".

Her approach seems to be a promising way to use quantum consciousness for therapy for some hard-to-treat psychiatric disorders.

Universal Geometric Entity

The Universal Geometric Entity (UGE)is the direct product of
the Universal Body (modelled from 120-dimensional ID)
and

the Universal Mind (modelled from 248-dimensional E8
which in turn is constructed from 256x256-dimensional Cl(16)).

Appendices: ID Character Table and E8 Representations

David Ford and John McKay wrote in the book "The Geometric Vein" (Springer-Verlag 1981):

"... The columns of the character tables of ... the binary icosahedral group ... [ID Icosahedral Double Group]... of order 120 are the (suitably normalized) eigenvectors of the Cartan matrices of type ... E8 ...

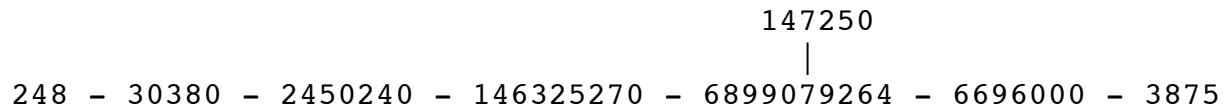
[Let $gr = (1/2) (- 1 - \sqrt{5})$ and $GR = (1/2) (- 1 + \sqrt{5})$ and note that $gr + GR = -1$]

...

1	1	1	1	1	1	1	1	1
2	-2	0	-1	1	GR	gr	-gr	-GR
2	-2	0	-1	1	gr	GR	-GR	-gr
3	3	-1	0	0	-gr	-GR	-GR	-gr
3	3	-1	0	0	-GR	-gr	-gr	-GR
4	4	0	1	1	-1	-1	-1	-1
4	-4	0	1	-1	-1	-1	1	1
5	5	1	-1	-1	0	0	0	0
6	-6	0	0	0	1	1	-1	-1

..."

The E8 Coxeter-Dynkin diagram, in which each vertex corresponds to a fundamental representation, is



E8 has 8 balance numbers which are 1,2,3,4,5,6,4,2,3
 Their product is 128 x 27 x 5 which multiplied by 8x7x6x5x4x3x2 gives the order 696,729,600 of the Weyl reflection group of E8

A Cartan matrix for E8 is

$$\begin{array}{cccccccc}
2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 2 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 2
\end{array}$$

Note that E8 can be constructed from the representations of E6 and D8.

The grade-1 vector representation of D8 is 120-dimensional.

The half-spinor representation of D8 is 128-dimensional.

The adjoint representation of E8 is $120 + 128 = 248$ -dimensional.

D8 and E6 both have trivial 1-dimensional scalar representations.

E6 has 27-dimensional and 78-dimensional representations.

The construction of E8 Dynkin representations

$$\begin{array}{cccccccc}
& & & & & & 147,250 & \\
& & & & & & | & \\
248 & - & 30,380 & - & 2,450,240 & - & 146,325,270 & - & 6,899,079,264 & - & 6,696,000 & - & 3,875
\end{array}$$

is as follows:

First, construct the exterior/wedge products of the E8 adjoint 248:

The grade-1 part has dimension 248.

The grade-2 part has dimension $248 \wedge 248 = 30,628$.

The grade-3 part has dimension $248 \wedge 248 \wedge 248 = 2,511,496$.

The grade-4 part has dimension $248 \wedge 248 \wedge 248 \wedge 248 = 153,829,130$.

The grade-5 part has dimension $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248 = 7,506,861,544$.

Now:

Keep the grade-1 part of dimension 248.

Subtract off 248

from $248^{\wedge}248 = 30,628$ to get 30,380.

Subtract off $2 \times 248^{\wedge}248 = 2 \times 30,628$

from $248^{\wedge}248^{\wedge}248 = 2,511,496$ to get 2,450,240.

Subtract off $2 \times 2,511,496$ and $2,450,240$ and $30,628$

from $248^{\wedge}248^{\wedge}248^{\wedge}248 = 153,829,130$ to get 146,325,270.

Subtract off $2 \times 153,829,130$ and $2 \times 146,325,270$

and $2 \times 2,511,496$ and $2,450,240$ and 248

from $248^{\wedge}248^{\wedge}248^{\wedge}248^{\wedge}248 = 7,506,861,544$ to get 6,899,079,264.

These are 5 of the 8 fundamental representations of E8.

They, like the D(N) and A(N) series constructions,

are all in the same exterior algebra (of $\wedge 248$),

and so can be represented as the vertices of a pentagon.

What about the 6th and 7th fundamental representations of E8?

Consider the 27-dimensional E6 representation space.

Add 32 copies of the 128-dimensional D8 half-spinor space,
and subtract off

one copy of the 248-dimensional E8 representation space
to get

a $27 + 32 \times 128 - 248 = 3,875$ -dimensional representation space.

Now, consider the antisymmetric exterior wedge algebra
of that 3,875-dimensional space.

The grade-1 part has dimension 3,875.

The grade-2 part has dimension $3,875 \wedge 3,875 = 7,505,875$.

Now:

Keep the grade-1 part of dimension 3,875.

Subtract off $5 \times 147,250$ and $2 \times 30,628$

and $3 \times 3,875$ and 3×248

from $3,875 \wedge 3,875 = 7,505,875$ to get 6,696,000.

They are the 6th and 7th fundamental representations of E8.

Since they are not in the same \wedge^{248} exterior algebra as
the 5 pentagon-vertex fundamental representations of E8,
they should not be vertices in the same plane as the pentagon.

However, since they are in the same $\wedge^{3,875}$ exterior algebra,
they should be collinear, one above and one below the pentagon,
thus forming a pentagonal bipyramid.

What about the 8th fundamental representations of E8?

Consider $2 \times 24 \times 24 - 1 = 2 \times 576 - 1 = 1,151$ copies of the 128-dimensional D8 half-spinor space, and subtract off one copy of the 78-dimensional E6 representation space to get a representation space of dimension $1,151 \times 128 - 78 = 147,328 - 78 = 147,250$.

Now, consider the antisymmetric exterior wedge algebra of that 147,250-dimensional space. The grade-1 part has dimension 147,250.

It is the 8th fundamental representation of E8. Since it is not in the same \wedge^{248} exterior algebra as the 5 pentagon-vertex fundamental representations of E8, it should not be a vertex in the same plane as the pentagon. Also, since it is not in the same $\wedge^{3,875}$ exterior algebra as the two bipyramid-peak-vertex fundamental representations of E8, it should not be a vertex on the same line as the pentagonal bipyramid axis. It should represent a vertex creating a triangle whose base is one of the sides of the pentagon and whose top is near one of the bipyramid-peak-vertices, to which it is connected by a line. To produce a symmetric figure, the vertex must be reproduced in 5 copies, one over each of the 5 sides of the pentagon. Then, for the entire figure to be symmetric, it must form an icosahedron. The binary icosahedral group $\{2,3,5\}$ is of order 120.

Another way to look at it is:

The graded sequence

248

$248 \wedge 248$

$248 \wedge 248 \wedge 248$

$248 \wedge 248 \wedge 248 \wedge 248$

$248 \wedge 248 \wedge 248 \wedge 248 \wedge 248$

has symmetry $Cy(5)$ of order 5 for cyclic permutations,

but since $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248$ is fixed

by its relation to $3,875 \wedge 3,875 \wedge 3,875$,

do not use Hodge duality on the 248 graded sequence.

The graded sequence

3,875

$3,875 \wedge 3,875$

$3,875 \wedge 3,875 \wedge 3,875$

has symmetry $Cy(3)$ of order 3 for cyclic permutations,

but since $3,875 \wedge 3,875 \wedge 3,875$ is fixed by

its relation to $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248$,

do not use Hodge duality on the 3,875 graded sequence.

The graded sequence

147,250

$147,250 \wedge 147,250$

has symmetry $Cy(2)$ of order 2 for cyclic permutations,

but since $147,250 \wedge 147,250$ is fixed by

its relation to $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248$,

do not use Hodge duality on the 147,250 graded sequence.

The +/- signs for the D5 half-spinors inherited

from E6 through E7

have symmetry of order 2.

Since the dimension of E_8 is $248 = 120 + 128$,
the sum of the 120-dimensional adjoint representation of D_8
plus
ONE of the 128-dimensional half-spinor representations of D_8 ,
there is a choice to be made as to
which of the two half-spinor representations of D_8 are used.
As they are mirror images of each other,
that choice has a symmetry of order 2.

Therefore:
the total symmetry group is of order $5 \times 3 \times 2 \times 2 \times 2 = 120$,
the symmetry of the binary icosahedral group $\{2,3,5\}$,
with 3 symmetries $Cy(5)$, $Cy(3)$, $Cy(2)$.

It corresponds, by the McKay correspondence, to the E_8 Lie Algebra.

What are the relations between 6,899,079,264
and $248^{248^{248^{248^{248}}}}$, $3,875^{3,875^{3,875}}$,
and $147,250^{147,250}$?

$$\begin{aligned}
6,899,079,264 &= 248^{248^{248^{248^{248}}} - 2 \times 248^{248^{248^{248}}} - \\
&\quad - 2 \times (248^{248^{248^{248}} - 2 \times 248^{248^{248}} - \\
&\quad\quad - (248^{248^{248} - 2 \times 248^{248}} - 248^{248}) - \\
&\quad - 2 \times 248^{248^{248}} \\
&\quad - (248^{248^{248} - 2 \times 248^{248}} - 248 = \\
&= 248^{248^{248^{248^{248}}} - \\
&\quad - 4 \times 248^{248^{248^{248}}} - \\
&\quad + 3 \times 248^{248^{248}} - \\
&\quad - 248 = \\
&= 7,506,861,544 - \\
&\quad - 4 \times 153,829,130 + \\
&\quad + 3 \times 2,511,496 - \\
&\quad - 248 = \\
&= 7,506,861,544 - \\
&\quad - 615,316,520 + \\
&\quad + 7,534,488 - \\
&\quad - 248 = 6,899,079,264
\end{aligned}$$

$$\begin{aligned}
6,899,079,264 &= 3,875 \wedge 3,875 \wedge 3,875 - 18 \times 248 \wedge 248 \wedge 248 - \\
&\quad - 3 \times 3,875 \wedge 3,875 + \\
&\quad + 3 \times 147,250 - \\
&\quad - 20 \times 248 - \\
&\quad - 6 \times 27 - 3 \times 8 = \\
&= 9,690,084,625 - 18 \times 153,829,130 - \\
&\quad - 3 \times 7,505,875 + \\
&\quad + 441,750 - \\
&\quad - 4,960 - \\
&\quad - 162 - 24 = \\
&= 9,690,084,625 - 2,768,924,340 - \\
&\quad - 22,517,625 + \\
&\quad + 441,750 - \\
&\quad - 4,960 - \\
&\quad - 162 - 24 = 6,899,079,264
\end{aligned}$$

$$\begin{aligned}
6,899,079,264 &= 147,250 \wedge 147,250 - 25 \times 248 \wedge 248 \wedge 248 - \\
&\quad - 38 \times 248 \wedge 248 \wedge 248 - \\
&\quad - 31 \times 248 \wedge 248 - \\
&\quad - 55 \times 248 - 128 - 27 = \\
&= 10,841,207,625 - 25 \times 153,829,130 - \\
&\quad - 38 \times 2,511,496 - \\
&\quad - 31 \times 30,628 - \\
&\quad - 55 \times 248 - 128 - 27 = \\
&= 10,841,207,625 - 3,845,728,250 - \\
&\quad - 95,436,848 - \\
&\quad - 949,468 - \\
&\quad - 13,640 - \\
&\quad - 155 = 6,899,079,264
\end{aligned}$$

In the relations, the 8 dimensional representation of D4 was also used.

J. F. Adams has written a paper entitled The Fundamental Representations of E8 published in Contemporary Mathematics, Volume 37, 1985, 1-10, and reprinted in The Selected Works of J. Frank Adams, Volume 2, edited by J. P. May and C. B. Thomas, Cambridge 1992, pp. 254-263.

The 8 fundamental E8 representations are

$$\begin{array}{cccccccc}
 & & & & & 147250 & & \\
 & & & & & | & & \\
 248 & - & 30380 & - & 2450240 & - & 146325270 & - & 6899079264 & - & 6696000 & - & 3875
 \end{array}$$

In his paper, Adams denotes the three at the ends as follows:

248 is denoted by alpha, which I will write here as a

3875 is denoted by beta, which I will write here as b

147250 is denoted by gamma, which I will write here as c .

Adams denotes the k th exterior power by λ^k which I will write here as \wedge^k and he denotes the k th symmetric power by σ^k which I will write here as S^k . Also, here I write (x) for tensor product.

J. F. Adams says:

"... we can allow ourselves to construct new representations from old by taking exterior powers, as well as tensor products and Z-linear combinations ...

... seven of the eight generators for the polynomial ring $R(E8)$ may be taken as

$$a, \wedge^2 a, \wedge^3 a, \wedge^4 a, b, \wedge^2 b, c.$$

The eighth may be taken either as $\wedge^5 a$, or as $\wedge^3 b$, or as $\wedge^2 c$.

The corresponding argument for D_n would say that one should begin by understanding three representations of $D_n = Spin(2n)$,

namely the usual representation on R^{2n} and the two half-spinor representations δ^+ , δ^- .

This we believe, so perhaps we can accept the analogue for E_8 .

...

we must get back to business and construct b and c .

...

To give explicit formulae we must agree on a coordinate system. The group E_8 contains a subgroup of type A_8 .

...

As a representation of A_8 , the Lie algebra $L(E_8)$ splits to give

$$L(E_8) = L(A_8) + \wedge^3 + \wedge^3^* .$$

Let e_1, e_2, \dots, e_9 be the standard basis in the vector space $V = C^9$ on which A_8 acts

...

We now introduce the element

$$v_{ik} = \sum_j (e_i^* e_j^* e_k^* (x) e_j e_k^* + e_j e_k^* (x) e_i^* e_j^* e_k^*)$$

in the symmetric square $S^2(a)$ in $(x) a$, where $a = L(E_8)$.

...

THEOREM 4.

(a) The representation $S^2(a)$ of E_8 contains a unique copy of b .

(b) This copy of b contains the elements v_{ik} .

(c) For $i \neq k$ the elements v_{ik} are eigenvectors corresponding to extreme weight of b .

(d) In particular (with our choice of details) v_{g1} is an eigenvector corresponding to the highest weight of b .

...

Theorem 4 allows us to realize b as the E_8 -submodule of $S^2(a)$ generated by v_{g1} (or any other v_{ik} with $i \neq k$.)

...

In fact, $S^2(a)$ contains a trivial summand ... and also an irreducible summand of highest weight ...

It turns out that the remaining summand has dimension 3,875,

which is precisely the dimension of b .

We now introduce the element

$$w_k = \sum_i e_i e_k^* (x) v_{ik}$$

$$= \sum_{(i,j)} e_i e_k^* (x) (e_i^* e_j^* e_k^* (x) e_j e_k^* + e_j e_k^* (x) e_i^* e_j^* e_k^*)$$

in $a(x) b$ in $a(x) S_2(a)$ in $a(x) a(x) a$.

...

THEOREM 5.

(a) The representation $a(x) b$ of E_8 contains a unique copy of c .

(b) This copy of c contains the elements w_k .

(c) The elements w_k are eigenvectors corresponding to the extreme weights of c .

(d) In particular (with our choice of details) w_1 is an eigenvector corresponding to the highest weight of c .

...

we may realize c as the E_8 -submodule of $a(x) b$ generated by w_1 (or any other w_k).

...

In fact, $a(x) b$ contains an irreducible summand of highest weight ... and the remaining summand has too small a dimension to contain two copies of c".

Adams then gives proofs of the theorems, involving such things as looking at the Lie algebra $L(E_8)$ as a representation of D_8 as which it splits to give $L(E_8) = L(D_8) + \text{delta-}$ where delta- is a half-spinor representation of D_8 .

One of the things that I think that I get out of Adams's paper is that it seems to me that in order to get 3875 and 147250 you have to look not only at the 248 representation of E_8 but also at representations of some subgroups of E_8 such as D_8 etc.

Here are some excerpts from 4 slides of a talk by David Vogan (which has some simplifications/omissions of technicalities) and from part of a www.liegroups.org web page

- **What's a Lie group?**
 - A continuous family of symmetries.
- **How many Lie groups are there?**
 - One for every regular polyhedron.
- **Which one is E_8 ?**
 - The one for the icosahedron.
- **What's a group representation?**
 - A way to change under symmetry.
- **What's a character table?**
 - A description of all the representations.

E_8 : 453,060 vertices \rightsquigarrow pieces of 240-dimensional flag variety.

How many Lie groups are there?

One for every regular polyhedron.



- 2D polygons: classical groups.
- Tetrahedron: E_6 , dimension 78.
- Octahedron: E_7 , dimension 133.
- Icosahedron: E_8 , dimension 248.

Which one is E_8 ?

The one for the icosahedron.

There are three different groups called E_8 , each one 248-dimensional and wonderfully complicated.

- **Compact E_8 .** Characters computed by Weyl in 1925.
In atlas shorthand, encoded by (1) .

(Which hides deep and wonderful work by Weyl.)

- **Quaternionic E_8 .** Characters computed in 2005.

In atlas shorthand, a 73410×73410 matrix. One entry:

$$3q^{13} + 30q^{12} + 190q^{11} + 682q^{10} + 1547q^9 + 2364q^8 + 2545q^7 \\ + 2031q^6 + 1237q^5 + 585q^4 + 216q^3 + 60q^2 + 11q + 1$$

- **Split E_8 .** This is the tough one.

complex group E_8 has three real forms: the compact form, the quaternionic form ($K=A_1 \times E_7$) and the split form ($K=D_8$). Each of these gives rise to a different kind of real group of type E_8 . Their blocks, and their sizes, are given by the `blocksizes` command of `atlas`:

	compact	quaternionic	split
compact	0	0	1
quaternionic	0	3,150	73,410
split	1	73,410	453,060

The last row means that the split group has 3 blocks (at infinitesimal character ρ), of sizes 1, 73,410 and 453,060 respectively. (The columns give the group on which the Vogan-dual block lives.)

Here is some material from www.aimath.org/E8/

“... **The size of the answer**

The result of the E_8 calculation is a matrix, or grid, with 453,060 rows and columns.

There are 205,263,363,600 entries in the matrix, each of which is a polynomial. The largest entry in the matrix is:

$$\begin{aligned}
 &152 q^{22} + 3,472 q^{21} + 38,791 q^{20} + 293,021 q^{19} + 1,370,892 q^{18} + 4,067,059 q^{17} \\
 &+ 7,964,012 q^{16} + 11,159,003 q^{15} + 11,808,808 q^{14} + 9,859,915 q^{13} + 6,778,956 q^{12} \\
 &+ 3,964,369 q^{11} + 2,015,441 q^{10} + 906,567 q^9 + 363,611 q^8 + 129,820 q^7 \\
 &+ 41,239 q^6 + 11,426 q^5 + 2,677 q^4 + 492 q^3 + 61 q^2 + 3 q
 \end{aligned}$$

If each entry was written in a one inch square, then the entire matrix would measure more than 7 miles on each side.

... The computation was completed on January 8, 2007.

Ultimately the computation took 77 hours of computer time, and **60 gigabytes to store the answer in a highly compressed form.**

... **By way of comparison, a human genome can be stored in less than one gigabyte.** For a more down to earth comparison, 60 gigabytes is enough to store 45 days of continuous music in MP3-format. ...



... Some other facts about the answer

Size of the matrix: 453,060

Number of distinct polynomials: 1,181,642,979

Number of coefficients in distinct polynomials: 13,721,641,221

Maximal coefficient: 11,808,808

Polynomial with the maximal coefficient: $152q^{22} + 3,472q^{21} + 38,791q^{20} + 293,021q^{19} + 1,370,892q^{18} + 4,067,059q^{17} + 7,964,012q^{16} + 11,159,003q^{15} + 11,808,808q^{14} + 9,859,915q^{13} + 6,778,956q^{12} + 3,964,369q^{11} + 2,015,441q^{10} + 906,567q^9 + 363,611q^8 + 129,820q^7 + 41,239q^6 + 11,426q^5 + 2,677q^4 + 492q^3 + 61q^2 + 3q$

Value of this polynomial at $q=1$: 60,779,787

Polynomial with the largest value at 1 which we've found so far: $1,583q^{22} + 18,668q^{21} + 127,878q^{20} + 604,872q^{19} + 2,040,844q^{18} + 4,880,797q^{17} + 8,470,080q^{16} + 11,143,777q^{15} + 11,467,297q^{14} + 9,503,114q^{13} + 6,554,446q^{12} + 3,862,269q^{11} + 1,979,443q^{10} + 896,537q^9 + 361,489q^8 + 129,510q^7 + 41,211q^6 + 11,425q^5 + 2,677q^4 + 492q^3 + 61q^2 + 3q$

Value of this polynomial at $q=1$: 62,098,473

...



P-adic Physics

P-adic Physics is useful in understanding the String Theory of R8 Universal Consciousness with interpretation of Strings as World-Lines of Many-Worlds.

At our scale, an elementary particles such as a lepton or quark is seen as a Vortex Cloud of virtual sea particle-antiparticle pairs whose objective form is that of a Kerr-Newman Black Hole whose identity is determined by a single Planck-scale Valence Particle whose past and future history path describes a World-Line.

As a 1-dim String World Line is perturbed (or vibrates) in the natural 26-dimensional space of Bosonic String Theory, it sweeps out a 2-dim String World Sheet, but the Line/Sheet structure breaks down at the Planck scale, at which scale P-adic structures emerge.

Lee Brekke and Peter G. O. Freund in Physics Reports 233 (1993) 1 - 66 say:
“... The boundary of the ordinary open string world sheet is the real line. Along with the usual open string, one can consider p-adic open strings whose world sheet has as boundary the p-adic line instead (the points on this boundary are labelled by p-adic numbers rather than real numbers). The world sheet itself is then no longer a continuous manifold but becomes a discrete homogeneous Bethe lattice, or Bruhat-Tits tree of incidence number $p + 1$. The p-adic strings thus correspond to specific discretizations of the world sheet. Studying all these discretizations (one for each prime p) together, as suggested by number theory, sheds new light on the ordinary string ...[and]...a new type of string, the adelic string. ...”.

B. Dragovich, A. Yu. Krennikov, S. V. Kozyrev, and I. V. Vololovich in 0904.4205 [math-ph] say:
“... Suppose we have a physical ... system and we make measurements. To describe results of the measurements, we can always use rationals. To do mathematical analysis, one needs a completion of the field Q of the rational numbers. ... there are only two kinds of completions of the rationals ... real R or p-adic Q_p number fields, where p is any prime number with corresponding p-adic norm $|x|_p$, which is non-Archimedean. ... One can argue that at the very small (Planck) scale the geometry of the spacetime should be non-Archimedean. ... Then rational, real or p-adic numbers should appear through a mechanism of number field symmetry breaking, similar to the Higgs mechanism. ...”.

Goran S. Djordjevic, Branko Dragovich, and Ljubisa Nestic in hep-th/0105030 say: "... the field of real numbers \mathbb{R} and the fields of p -adic numbers \mathbb{Q}_p exhaust all number fields which may be obtained by completion of ... the field of rational numbers \mathbb{Q} , and which contain \mathbb{Q} as a dense subfield. ...

Real and p -adic numbers are unified in the form of adèles. An adèle x is an infinite sequence $x = (x_0, x_2, \dots, x_p, \dots)$, where x_0 is in \mathbb{R} and x_p are in \mathbb{Q}_p with the restriction that for all but a finite set S of primes p one has x_p in \mathbb{Z}_p , where $\mathbb{Z}_p = \{a \in \mathbb{Q}_p : |a|_p \leq 1\}$ is the ring of p -adic integers.

Componentwise addition and multiplication are natural operations on the ring of adèles A , which can be regarded as

$$A = \bigcup_S A(S), \quad A(S) = \mathbb{R} \times \prod_{p \in S} \mathbb{Q}_p \times \prod_{p \notin S} \mathbb{Z}_p$$

...

As adèles contain real and p -adic numbers, and consequently archimedean and nonarchimedean geometries, it has been natural to formulate adelic quantum mechanics, which provides a suitable framework for a systematic investigation of the above question. Uncertainty in space measurements

$$\Delta x \Delta y \geq \Delta L^2 = \hbar G / c^3 = 10^{(-66)} \text{ cm}^2$$

which comes from ordinary quantum gravity considerations based on real numbers, restricts application of \mathbb{R} and gives rise for employment of \mathbb{Q}_p at the Planck scale. It is also reasonable to expect that fundamental physical laws are invariant under interchange of \mathbb{R} and \mathbb{Q}_p

...

one can introduce adelic path integral as an infinite product of ordinary and p -adic path integrals for all primes p . Intuitively, we regard adelic Feynman's functional integrals as path integrals on adelic spaces. ...

the formalism of ordinary and p -adic path integrals can be regarded as the same at different levels of evaluation, and the obtained results have the same form.

In fact, this property of number field invariance has to be natural for general mathematical methods in physics and fundamental physical laws.

In the present case it is mainly a consequence of the form invariance of the Gauss integral under interchange of \mathbb{R} and \mathbb{Q}_p ...".

So, at the Planck scale,

p -adic tree structures describe the branching of the Worlds of the Many-Worlds and the non-loop terms of the Feynman Sum-Over-Histories Path Integral and p -adic non-zero genus structures describe Path Integral loops.

Chekhov, Mironov, and Zabrodin in Comm. Math. Phys. 125 (1989) 675-711
http://projecteuclid.org/DPubS/Repository/1.0/Disseminate?view=body&id=pdf_1&handle=euclid.cmp/1104179635

discuss P-adic geometry and Trees in the context of String Theory.

They say "... we propose a multiloop generalization ...

We consider the Bruhat-Tits tree which is an infinite homogeneous graph without cycles, each vertex being connected with exactly $p + 1$ neighbours by edges of unit length, as p-adic zero genus Riemann surface. ...

To produce p-adic Riemann surfaces of higher genera we should factorize the tree by some discrete (Schottky) groups. The surface obtained is the graph with cycles (their number is equal to the genus of the surface), the properties of this graph can be described by means of the ... reduced graph, which is the finite subgraph containing only the cycles with crosspieces between them. ...

2.2. Riemann Surface Over \mathbb{Q}_p and Bruhat-Tits Tree

... describe the construction of the Riemann surfaces [RS] over \mathbb{Q}_p field.

To obtain the extended zero genus \mathbb{Q}_p RS one has to factorize $\text{PGL}(2, \mathbb{Q}_p)$ by its maximal compact subgroup, that is $\text{PGL}(2, \mathbb{Z}_p)$, being determined as 2×2 matrices with p-adic integer entries and invertible determinant in \mathbb{Z}_p . It is this homogeneous space that is called the Bruhat-Tits tree T . It is ... the connected infinite graph with no loops each vertex of T being connected with $p + 1$ neighbour vertices by edges. Obviously, any two vertices z_1, z_2 in the tree are connected by exactly one path $z_1 \rightarrow z_2$. We define the distance $d(z_1, z_2)$ between these vertices to be the number of edges in the path $z_1 \rightarrow z_2$. There are half-axes in the tree which are infinite subtrees with no branch points but with a single starting point (Fig. 2).

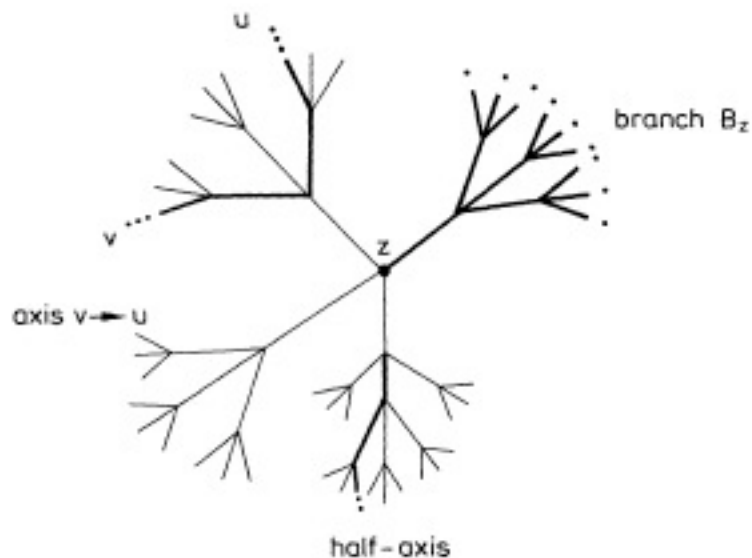


Fig. 2. Subgraphs in the tree: half-axis, axis $v \rightarrow u$ and branch B_z

equivalence classes ... for half-axes ... [are called] ... rays.

... the tree T can be compactified by adding the set of "infinitely far points" dT defined as the set of all rays.

In fact, dT can be canonically identified with $P^1(Q_p)$.

On the other hand,

the $PGL(2, Q_p)$ -action can be naturally extended to dT from the tree T , so we shall consider dT as the boundary of a compactified tree $T \cup dT$ with $PGL(2, Q_p)$ -action on it.

In order to "coordinatize" $P^1(Q_p)$ we fix a point C (the "origin") in T .

This vertex corresponds to three half-axes starting at C whose endpoints in $dT = P^1(Q_p)$ are $(0, 1, \infty)$, by definition.

Then we can identify $P^1(Q_p)$ with Q_p and after fixing C only $PGL(2, Z_p)$ -freedom remains ...

C is the fixed point of $PGL(2, Z_p)$ and we describe the $PGL(2, Q_p)$ -action on T manifestly as follows: $PGL(2, Q_p)$ acts on $T = PGL(2, Q_p) / PGL(2, Z_p)$

transitively and isometrically (i.e. the distances $d(\cdot)$ are conserved);

vertices correspond to $g PGL(2, Z_p) g^{-1}$ subgroups, g in $PGL(2, Q_p)$,

and edges correspond to $g H g^{-1}$ subgroups, $H = 2 \times 2$ matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

for a, b, c, d in Z_p and $ad - bc \notin M$,

where M is the unique maximal ideal in the ring Z_p

(i.e. $M = \{ x \in Q_p \mid |x|_p < 1 \}$).

Let us define a branch B_z to be an entire subgraph of T with the only boundary point z in the interior of T . The graph B is called entire if $B \setminus dB$ is a connected graph (Fig. 2) ...

we assume that the branches contain no cycles ... Then the set of rays contained in B_z corresponds to an open domain dB_z in dT and induce a natural topology on Q_p . Thus we have described the zero genus EQ_pRS .

...

Now introduce Q_pRS and EQ_pRS of higher genera.

We consider a Schottky group Y over Q_p acting on T .

Then for any hyperbolic element y in Y the only y -invariant axis in T exists which is called the y -axis

(by definition, an axis is an infinite connected subtree with no branch vertices and terminating vertices inside T (Fig. 2)).

Each element y acts by shifts along the corresponding y -axis, and for any y conjugated to the element $\begin{pmatrix} q & 0 \\ 0 & 1 \end{pmatrix}$ $0 < |q|_p = p^{-\text{ord}_p(q)} < 1$, the shift is equal to $\text{ord}_p(q)$. So the hyperbolic element has no invariant vertices and edges. ...

the action of y on the whole tree is the shift along y axes defined by $|q|_p$

with a simultaneous "rotation" around y-axes defined by the "phase" of q , i.e. by $q \bmod p$.

We define the Schottky tree $T(Y)$, which is a union of axes of all elements of Y and crosspieces between them (crosspiece is the unique finite path which has common vertices but not edges with two chosen axes), or, equivalently, a minimal connected subgraph containing the axes of all elements of Y ... Then $T(Y)$ is in T , $dT(Y)$ is in dT and $T(Y)/Y = FR$ is a finite graph, which is called the reduced graph ...

the reduced graph permits us to construct $EQpRS$ from the Bruhat-Tits tree.

As the first step we consider the simplest example, namely, the torus.

We choose an element y generating Y_1 in the form: $y = \begin{pmatrix} q & 0 \\ 0 & 1 \end{pmatrix}$, $0 < |q|_p < 1$

The y -axis passes through the origin C , the fixed points u, v in $Qp = dT$ are zero and infinity and any element of Y_1 acts on the axis $oo \rightarrow 0$ by d -shifts, $d = 0 \bmod(\text{ord}_p(q))$ (see Fig. 3a).

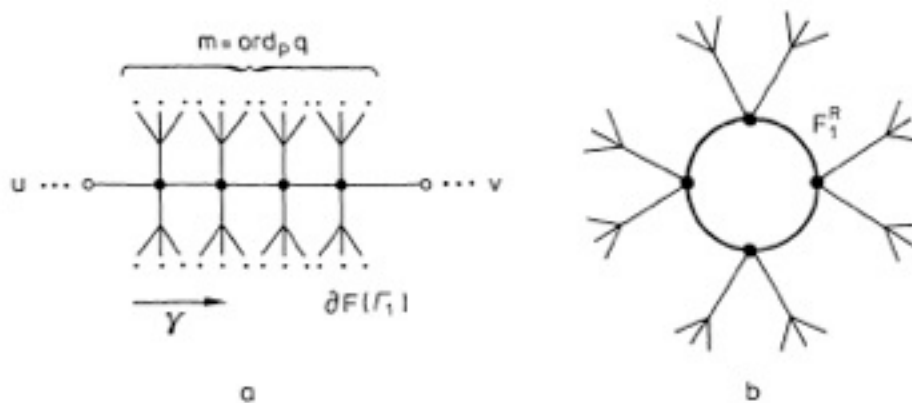


Fig 3 a-b. A typical fundamental domain of the Schottky group Γ_1 in the tree. **b.** The "extended" p -adic torus (in the case of $p=3$)

So the Schottky tree is a single axis,

FR_1 is a cycle (ring) consisting of $m = \text{ord}_p(q)$ edges.

To obtain $EQpRS$ we factorize T by $T_1 : F_1 = T / Y_1$ (Fig. 3b).

The result may be realized as a fundamental domain for Y_γ glued into a ring.

Correspondingly, it induces the isomorphism $dFF_1 = d(T/Y_1) = dT / Y_1$.

So far we have seen that FR_1 can be produced from F_1 by truncating all branches with origins at the reduced graph. It appears to be the general procedure for the surface of arbitrary genus. The inverse operation is clear as well:

to construct $EQpRS$ $F_g = T/Y_g$ it is necessary to draw the reduced graph with a given number of loops (this number g is equal to genus of the surface) and after that to add all the necessary branches with origins at the vertices

of the reduced graph in an evident way (Fig. 4a and b).

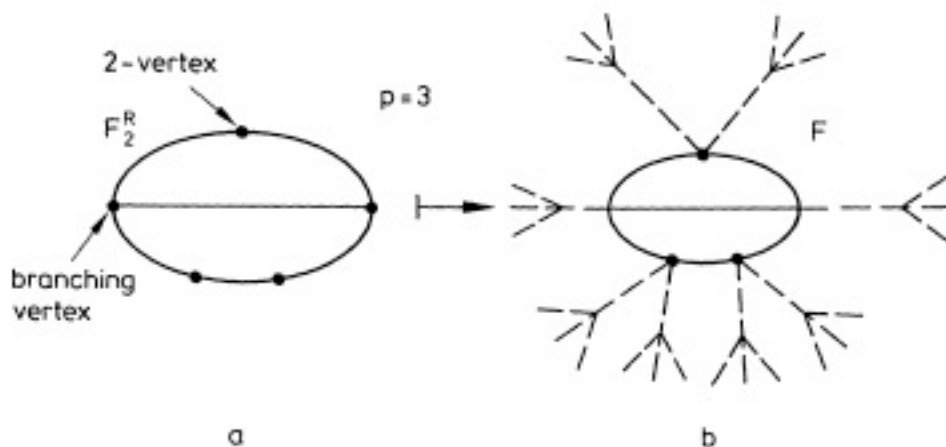


Fig. 4 a-b. A reduced graph for $g=2$. **b.** The whole factorized tree ($g=2, p=3$)

If one treats identity transformation as the trivial Schottky group Y_0 , then FQ is merely a single vertex (denoted above as C), and $T/Y_0 = T$. Now one can see that the reduced graph consists of $3g - 3$ or less segments [segment in FR_g is the line containing only 2-vertices and connecting two branching vertices in FR_g (see Fig. 4a)]. We denote s_i the lengths of these segments. In fact, they are "the moduli" of the corresponding p -adic surface. Strictly speaking, these parameters are p -adic orders of the moduli of an algebraic curve, but we shall call them merely moduli. Thus FR_g provides a good description of the moduli space of Riemann surfaces over \mathbb{Q}_p . . .".

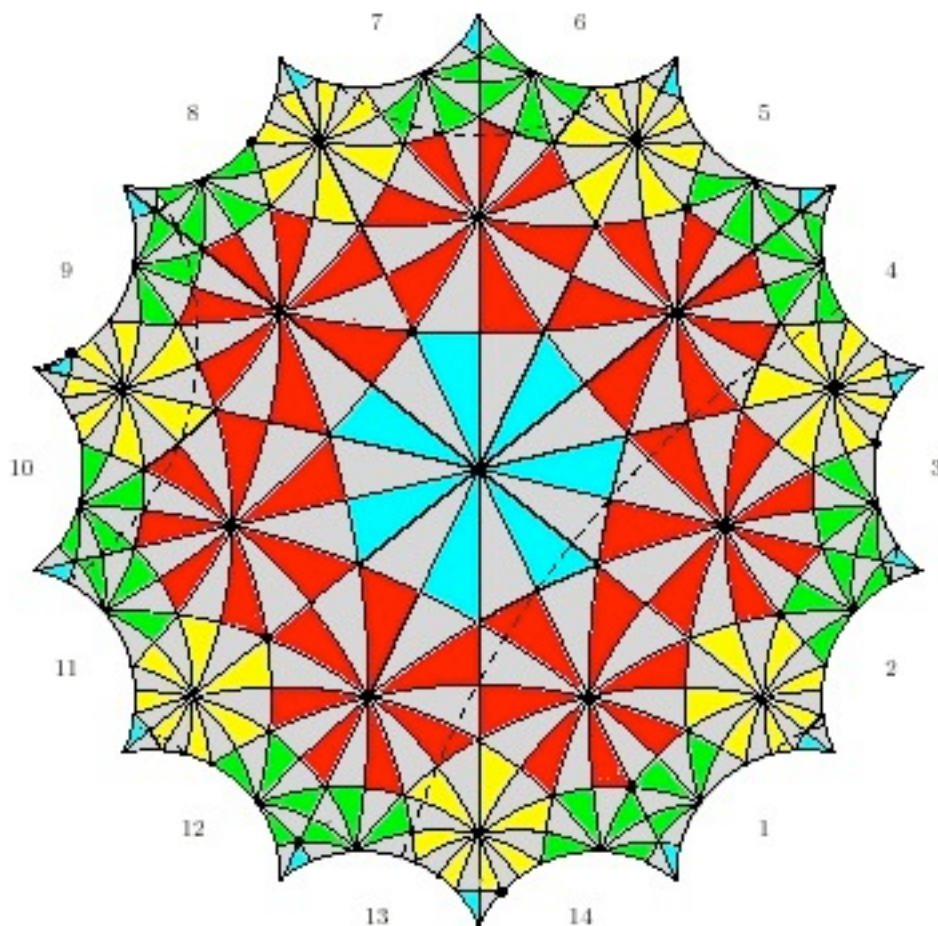
It is interesting to have intuitive visualization of the p -adic tree structures that are fundamental for the physics (including the World-Line String Theory that gives the Bohm Quantum Potential of the Universal Consciousness) at the Planck scale.

Anthony Phan has a web page at <http://www-math.univ-poitiers.fr/%7Ephan/exemples2.html> in which he says "... Paul Broussous and I had some discussions ... he showed me how to have a visual perception of p -adic fields \mathbb{Q}_p (where p is some prime number): it may be represented thru some graphical representation of $P^1(\mathbb{Q}_p)$ which is this field compactified by a point at infinity, itself being the boundary of some infinite tree. ...".

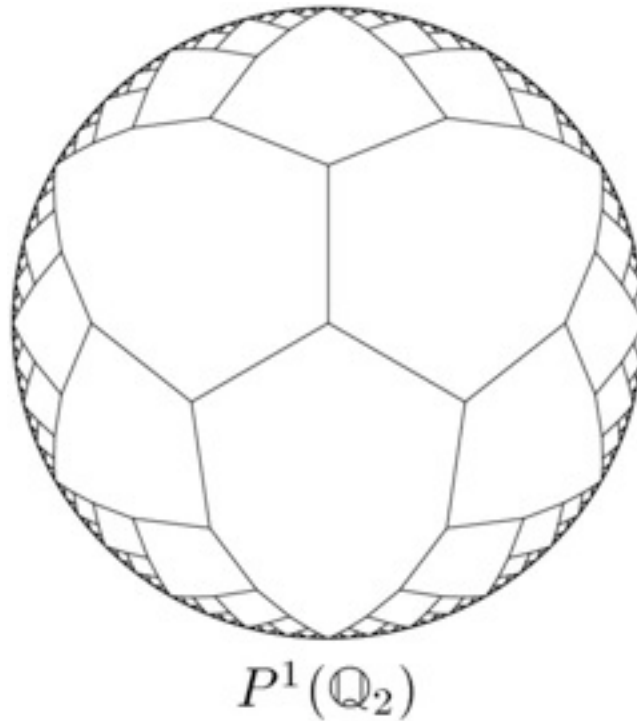
Paul Broussous has a web page at <http://www-math.univ-poitiers.fr/~broussou/recherche.htm> in which he says in part (this is a google automated translation from French with some modification by me)

"... In the same way that one can visualize the projective line over the field of real numbers as a circle, the projective line on the field \mathbb{Q}_p of p-adic occurs naturally as the set of "pieces" of a uniform tree of valence $p + 1$ ".

The idea is that the p-adic tree starts in the center of a circle and branches out toward the edge of the circle, with the edge of the circle being considered to be infinity as is done in hyperbolic geometry of the Poincare disk, an example being the Klein Configuration



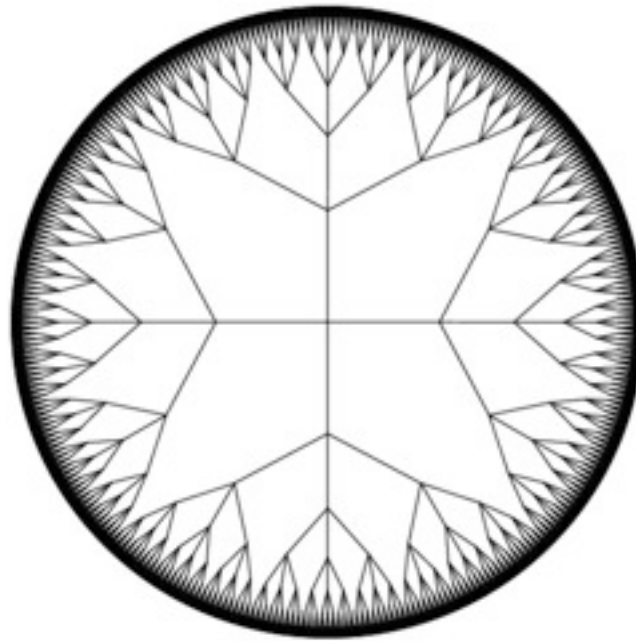
For the prime $p = 2$ the Broussous method gives



which as you can see has $p+1 = 2+1 = 3$ trees from the center with 2-fold branching to the edge of the circle (infinity in the hyperbolic Poincare Plane type projection).

The binary 2-fold branching is like the YiJing
and
the binary + and - charges of electromagnetism
and
the binary system of fermion particles and antiparticles
and
the geometry of hypercubes.

For the prime $p = 3$ the Broussous method gives



$$P^1(Q_3)$$

which has $p+1 = 3+1 = 4$ trees from the center with 3-fold branching to the edge of the circle (infinity in the hyperbolic Poincare Plane type projection).

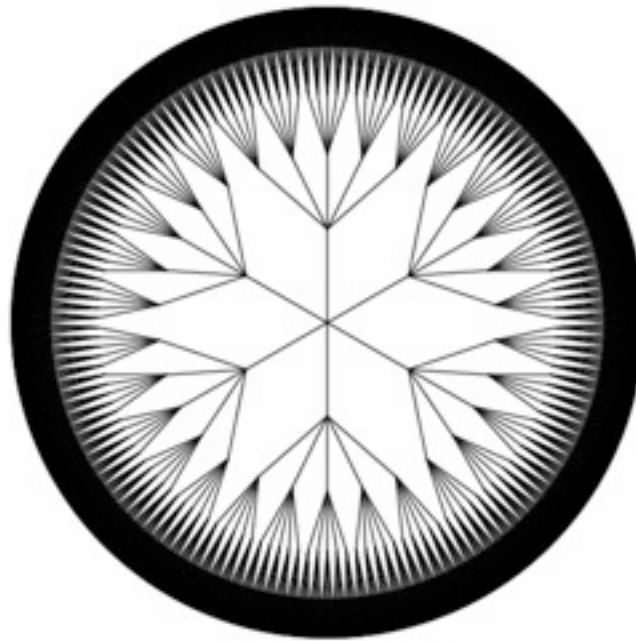
3-fold branching may correspond to such things as:
ternary sedenions (16-dim numbers multiplied 3 at a time);
the geometry of sub-hypercubes of a hypercube;
and

the Tai Hsuan Ching of $3 \times 3 \times 3 \times 3 = 81$ tetragrams of ternary lines

I have web material about it at

<http://www.valdostamuseum.org/hamsmith/ichgene6.html#TaiHsuanChing>

For the prime $p = 5$ the Broussous method gives



$$P^1(\mathbb{Q}_5)$$

which has $p+1 = 5+1 = 6$ trees from the center with 5-fold branching to the edge of the circle (infinity in the hyperbolic Poincaré Plane type projection). 5-fold branching may correspond to such things as: the icosahedron.

Note that these primes 2,3,5 were all important in the AMS Notices paper "The p-adic Icosahedron"

http://www.staff.science.uu.nl/~corne102/publications/p-adic_icosahedron.pdf

which has similar pictures, and goes on to say:

"... Projecting the icosahedron from its center to a circumscribed sphere maps each edge onto a part of a geodesic line on the sphere.

These geodesic lines intersect at vertices, midpoints of edges, or barycenters of faces.

They provide a tessellation of the sphere by 120 triangles with angles $(\pi/2, \pi/3, \pi/5)$ (Figure 3).

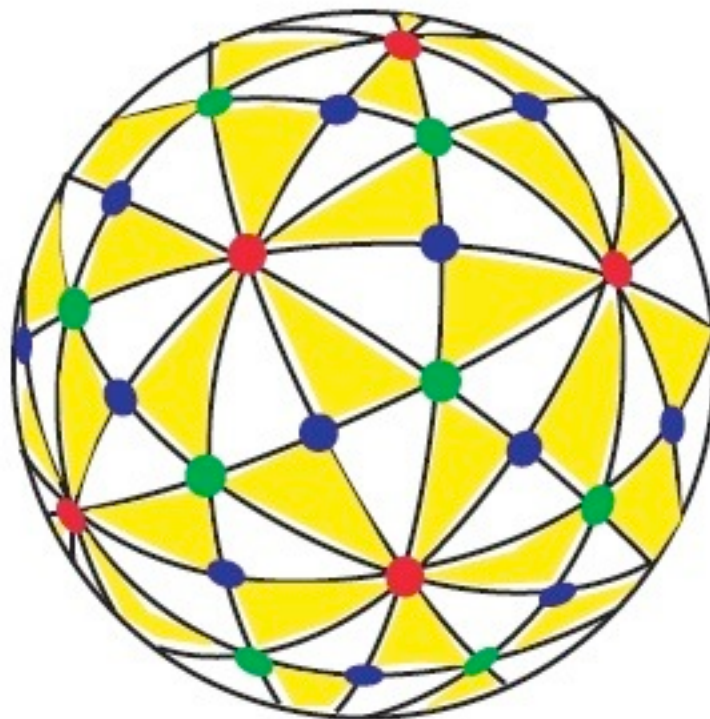


Figure 3. Icosahedral tessellation of the Riemann sphere.

... Let A_5 denote the group of even permutations on five letters, i.e., the simple group with 60 elements.

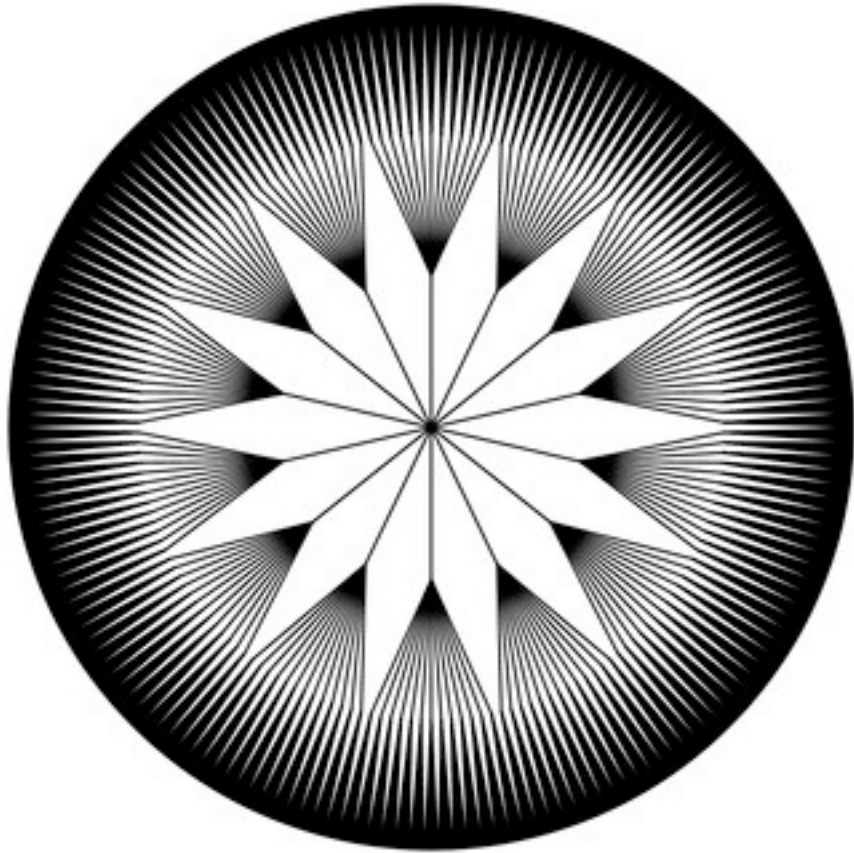
An icosahedral group is a copy of A_5 embedded in $PGL(2, \mathbb{C})$, the automorphism group of the Riemann sphere (= complex fractional linear transformations).

...

In order to find the p-adic analogue of the icosahedron ...
we have to introduce the p-adic analogue of the Riemann sphere,
which is an analytic structure on the projective line P^1 . The naive way
of "doing analysis with the p-adic metric on the coordinates" does not work
(because of total disconnectedness). ...
we will content ourselves with a description at the level of trees ...
The Bruhat-Tits tree T of $PGL(2, \mathbb{Q}_p)$... edges emanating from any given vertex
are in one-to-one correspondence with \mathbb{F}_p -rational points of P^1 .
The graph T is a regular $(p + 1)$ -valent tree ...".

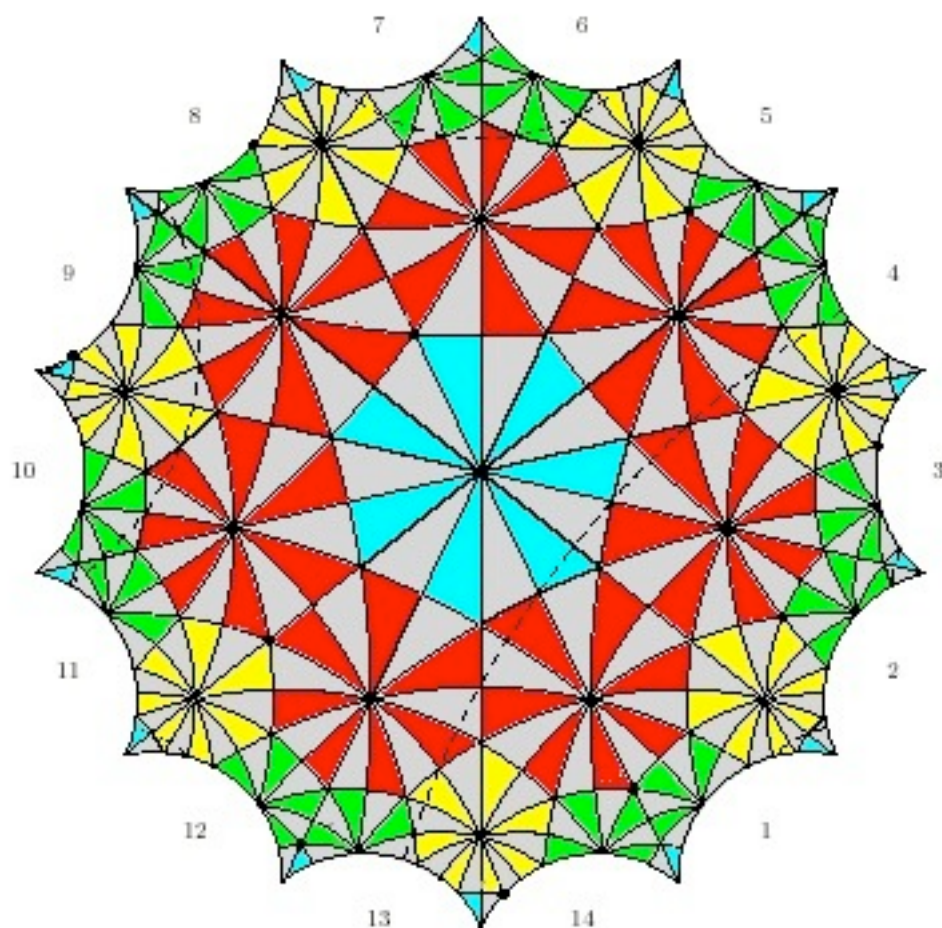
Notice that the icosahedral structure gives a correspondence between
the 120 triangles of the sphere and the 120 vertices of the 600-cell
and
that the icosahedral finite subgroup structure of the Riemann sphere
is similar to
the McKay correspondence between the icosahedral double group subgroup
structure of the 3-sphere and the E_8 Lie algebra.

For the prime $p = 13$ the Broussous method gives



which has $p+1 = 13+1 = 14$ trees from the center with 13-fold branching to the edge of the circle (infinity in the hyperbolic Poincare Plane type projection).

The 14 trees seem to correspond to the 14 sections of the Klein Configuration



which can be interpreted as describing the same physics model that I usually describe in terms of E8 or Clifford Algebra. I have a paper about that at

<http://www.valdostamuseum.org/hamsmith/KleinQP.pdf>

but

here I will just say that its Riemann surface is a 168-sheet covering of the sphere and so is related to the 168-element group $PSL(2,7)$ which is a symmetry group of Octonion multiplications and Octonions are basic for E8.

13 is the only number between 5 and 89 that is both Prime and Fibonacci.

David C. Terr wrote a paper "Fibonacci Expansions and F-Adic Integers"

<http://www.fq.math.ca/Scanned/34-2/terr.pdf>

in which he applies the ideas of p-adic (prime) theory

to his new idea of F-adic (Fibonacci) theory,

which should be of interest for 13 since it is both prime and Fibonacci,

as are 2, 3, and 5 of the icosahedral structures.

He says

"... The Fibonacci expansion of nonnegative integers ... may be thought of as a base-T expansion, where $T = (1/2)(1 + \sqrt{5}) = 1.61803$ is the golden mean).

...

the p-adic integers form a ring, but [some of] the F-adic integers do not.

My main result in this paper is that there is a 1-1 correspondence between the F-adic integers and the points on a circle ... of circumference T^2 ...",

so

it is interesting that 13 (through its Fibonacci aspect) is related to the Golden Ratio (similar to 2, 3, 5 and icosahedral structure).