Frank Dodd (Tony) Smith, Jr. - 2010

E8 Universe in a Molecular N-chain

An experiment with N-chains of molecules has shown Emergent E8 symmetry

that shows the Origin of the E8 symmetry of the E8 Physics model that describes the Origin of our Universe.

This paper has 3 parts:

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1 - Molecular N-chain experiment shows E8 symmetry

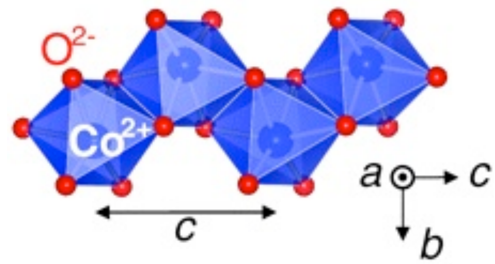
In their paper

"Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E8 Symmetry"

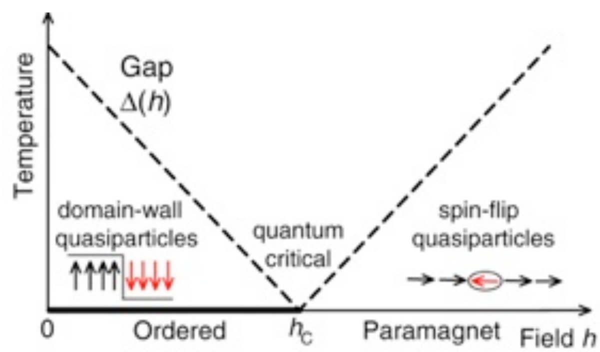
(Science 327 (8 January 2010) 177-180) R. Coldea, D. A. Tennant, E. M. Wheeler, E. Wawrzynska, D. Prabhakaran, M. Telling, K. Habicht, P. Smeibidl, K. Kiefer say:

"... Quantum phase transitions take place between distinct phases of matter at zero temperature. Near the transition point, exotic quantum symmetries can emerge that govern the excitation spectrum of the system. ... the simplest of systems, the Ising chain, promises a very complex symmetry, described mathematically by the E8 Lie group ... We realize this system experimentally by using strong transverse magnetic fields to tune the quasi&endash;one-dimensional Ising ferromagnet $CoNb_2O_6$ (cobalt niobate) through its critical point. ... Just below the critical field, the spin dynamics shows a fine structure with two sharp modes at low energies, in a ratio that approaches the golden mean predicted for the first two meson particles of the E8 spectrum. ...

the insulating quasi-1D Ising ferromagnet $CoNb_2O_6$ ...[has]... magnetic Co^{2+} ions ... arranged into near-isolated zigzag chains along the c axis ... Large single crystals can be grown ...

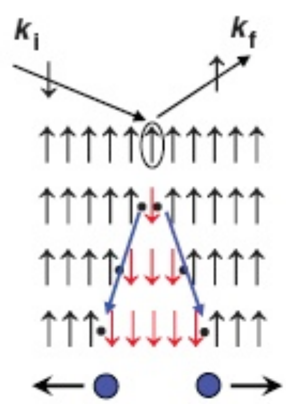


... The Ising exchange J favors spontaneous magnetic order along the z axis ... whereas the transverse field h forces the spins to point along the perpendicular +x direction ... This competition leads to two distinct phases, magnetically ordered and quantum paramagnetic, separated by a continuous transition at the critical field hC = J/2 ...

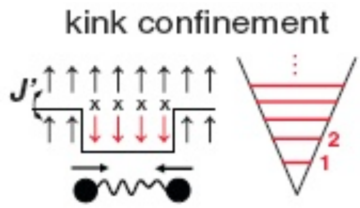


... Qualitatively, the magnetic field stimulates quantum tunneling processes between ... [up and down]... spin states and these zero-point quantum fluctuations "melt" the magnetic order at hC ...

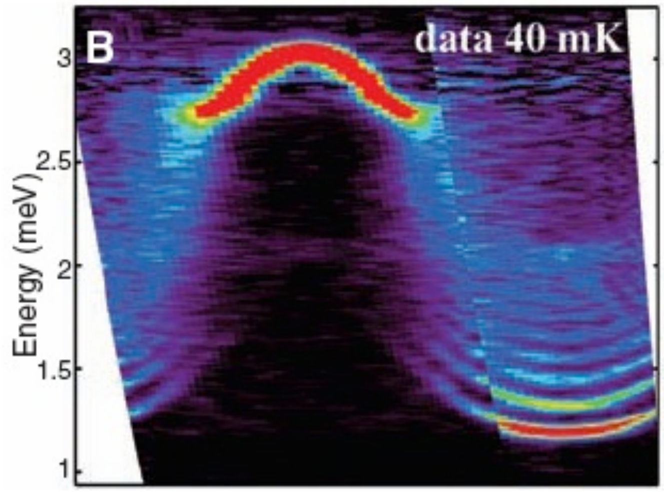
Neutrons scatter by creating a pair of kinks ...



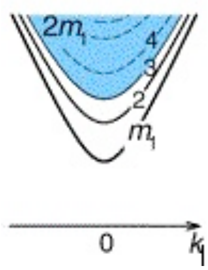
... In the ordered phase, kink separation costs energy as it breaks interchain bonds ... leading to an effective linear "string tension" that confines kinks into bound states ...



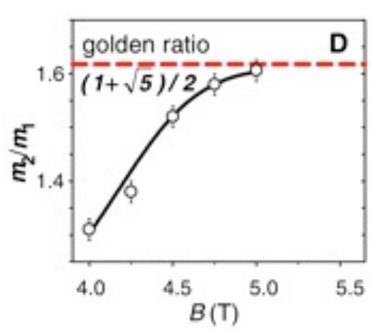
...Expected excitations ... consist of ... pairs of kinks ... below ... a critical field BC = 5.5 T .... The continuum splits into a Zeeman ladder of two-kink bound states deep in the ordered phase. ...



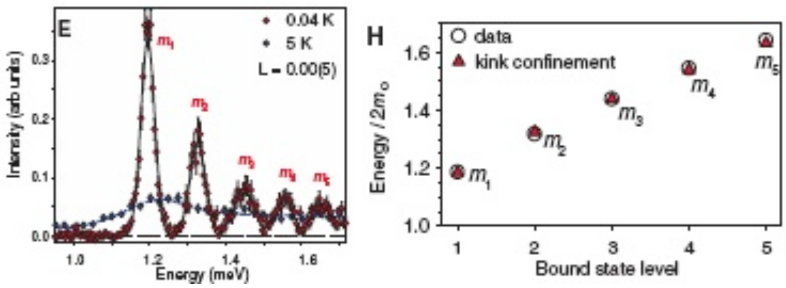
... Lines indicate bound states; shaded area is the 2m1 continuum ...



... The ratio m2/m1 approaches the golden ratio (dashed line) just below the critical field ...



... The kinks are a crucial aspect of the physics in one dimension, and their spectrum of confinement bound states near the transition field will be directly related to the low-energy symmetry of the critical point. ... If the 1D Ising chain is precisely at its critical point (h = hC), then the bound states stabilized by the additional longitudinal field hz morph into the "quantum resonances" that are a characteristic fingerprint of the emergent symmetries near the quantum critical point. ... Zamolodchikov proposed precisely eight "meson" bound states (the kinks playing the role of quarks), with energies in specific ratios given by a representation of the E8 exceptional Lie group ... At least five modes can be clearly observed ...

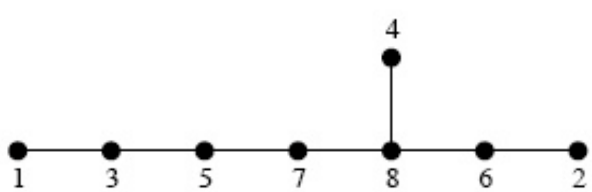


...".

Using material from the paper by Uwe Grimm and Bernard Nienhuis (arXiv hep-th/9610003) the 8 bound states of Zamolodchikov have the properties:

j		mj/m1	
1	oxoooooo	1	= 1
2	oxxooooo	1.618034...	= 2 cos (6 pi / 30)
3	oxxxoooo	1.989044...	= 2 cos (pi / 30)
4	oxxxxooo	2.404867...	= 4 cos (6 pi / 30) cos(7 pi / 30)
5	oxxxxxoo	2.956295...	= 4 cos (6 pi / 30) cos(4 pi / 30)
6	oxxxxxxo	3.218340...	= 4 cos (6 pi / 30) cos(pi / 30)
7	oxxxxxxx	3.891157...	= 8 cos^2(6 pi / 30) cos(7 pi / 30)
8	xxxxxxxx	4.783386...	= 8 cos^2(6 pi / 30) cos(4 pi / 30)

where the E8 bound state numbers j correspond to vertices on the E8 Dynkin diagram



As to the E8 exponents, according to the book "Reflection Groups and Coxeter Groups" by James E. Humphreys (Cambridge 1990) are the degrees of the polynomial invariants of the Weyl group of E8

and E8 has degrees 2, 8, 12, 14, 18, 20, 24, 30 and the product of the E8 degrees is 2^14 x 3^5 x 5^2 x 7 which is the order of the Weyl group of E8

and the E8 exponents are each just one less than each of the E8 degrees, that is, 1, 7, 11, 13, 17, 19, 23, 29

Patrick Dorey in his paper "Hidden geometrical structures in integrable models" (arXiv hep-th/9212143) said:

"... Zamolodchikov was able to extract much non-perturbative information about ... the scaling region of the Ising model in a magnetic field ...[for which]... [a]t criticality, the continuum limit of the model is well-known to be described by a c = 1/2 conformal field theory ... essentially because the perturbative expansions for certain quantities of interest truncate after finitely many terms. In particular, his 'counting argument' led to the conclusion that both epsilon and sigma perturbations of the Ising model preserve certain higher-spin conserved charges, and hence are integrable.

For the sigma perturbation, these spins are the numbers 1, 7, 11, 13, 17, 19, 23, 29, repeated modulo 30. ...

The critical Ising model being the simplest unitary conformal field theory, one might expect its integrable deformations to have S-matrices of this simplest possible form. Indeed, at T =/= Tc (the epsilon perturbation) the model is a theory of free massive fermions, which certainly have a diagonal S-matrix. Perturbing by sigma, things are not quite so simple, but in fact the S-matrix is again diagonal. A sequence of ingenious arguments, combining general principles with certain physical inputs specific to the Ising model, led Zamolodchikov to propose ...[an]... expression for the S-matrix element for the scattering of two of the lightest particles, as a function of their relative rapidity theta

S11 = - (2) (10) (12) (18) (20) (28)

where ... (x) = sinh(theta / 2 + i pi x / 60) / sinh(theta / 2 + i pi x / 60)

... How to proceed from here? The key notion is that of a bootstrap equation ... For the 1 1 -> 2 fusing, the bootstrap equation predicts

S12(theta) = S11(theta - i pi / 5) S11(theta + i pi / 5) = (6) (8) (12) (14) (16) (18) (22) (24)

while 1 1 -> 3 yields

S13(theta) = S11(theta - i pi / 30) S11(theta + i pi / 30) =

= (1) (3) (9) (11)^2 (13) (17) (19)^2 (21) (27) (29)

... there are ... further dodgy points ... The bootstrap equation is therefore best taken as a 'working axiom', to be re-examined in the event that it contradicts other information. For the case in hand, there don't seem to be any problems. ... Now iterate! The procedure just outlined is sufficiently well-defined that there is nothing (apart, again, from growing tedium) to prevent its continuation, resulting in an increasing collection of particles, S-matrix elements and nonvanishing three-point couplings, all bound together by the bootstrap equations. But it is worth persevering: as Zamolodchikov discovered, quite remarkably the process closes in on itself after a total of eight particle types have been encountered. ...

However, all this turns out to be unnecessary. The iteration of the bootstrap equations for the spin-perturbed Ising model is simply a more complicated way of doing the Weyl group computation presented in the first half of this section, while the fact that these equations are implied by the pole structure of the very functions that they constrain, when viewed from this perspective, follows from simple properties of the E8 root system.

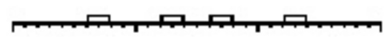
Some sort of connection was perhaps to have been expected, given that (a) the c=1/2 conformal field theory can be realised as the coset model E8^1 x E8^1 / E8^2 within which the field sigma corresponds to the branching (id, id, adj) ; (b) the spins of the conserved charges given earlier are precisely the exponents of E8, repeated modulo the Coxeter number; and (c) the masses m1, m2, ... , m8 can be formed into the Perron-Frobenius eigenvector for the incidence matrix of the E8 Dynkin diagram ...

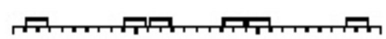
The key is to rewrite the S-matrix elements in a slightly different way ... by introducing a new, larger , building block

$$\{x\} = (x - 1) (x + 1)$$

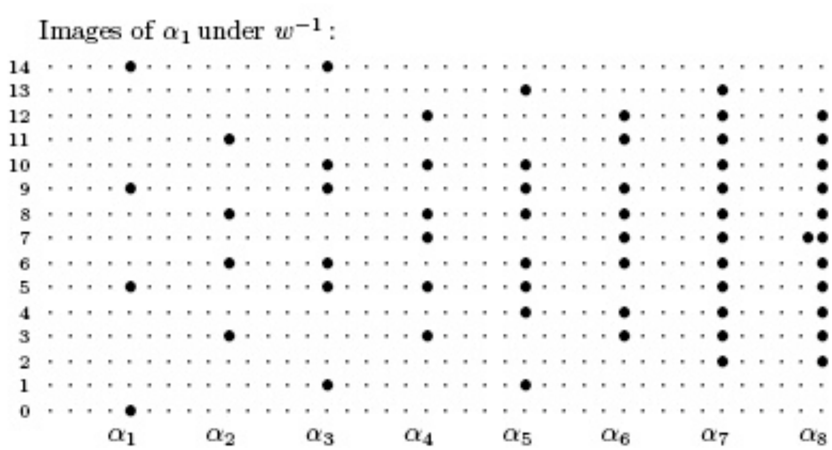
The earlier expressions become:

S11 = {1} {11} {19} {29} = 

S12 = {7} {13} {17} {23} = 

S13 = {2} {10} {12} {18} {20} {28} = 

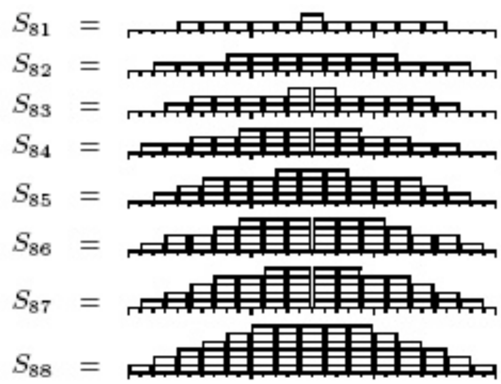
The 'brick wall' notation for S-matrix elements ... represents each factor {x} in a product by a brick , centered at x. The connection with the Weyl group ... can be seen by taking these three pieces of wall, rotating them by 90 degrees, and comparing them with the first three columns of ...



... w = r3 r4 46 r7 r1 r2 r5 r8 ...[is]... a Coxeter element ... of the Weyl group ... in a Steinberg ordering ... the action of a simple reflection ra on any simple root ab is just ra ab = a_b - C_ba a_a

...[where]... C_ab = 2 a_a . a_b / a_b^2 [is]... the Cartan matrix ... In terms of the Dynkin diagram, ra negates a_a , adds a_a to each a_b joined to a_a by a link, and leaves all the other simple roots unchanged. ... The sequence ...[w^-1 , w^-2, w^-3 and so on]... must ultimately repeat ... after 30 steps ... The first 14 of these steps are shown in the ...[above]... table ... there was no need to invent a notation for the negative of a simple root ... as all the roots from 0 to 14 were positive-linear combinations of the simple roots ...

the scattering amplitudes of the heaviest particle, 8, can also be checked:

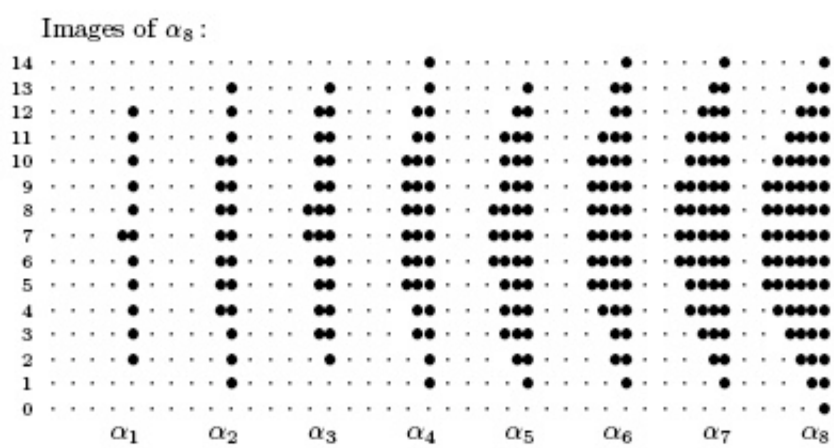


...[for example, S88 =

{1} {3}^2 {5}^3 {7}^4 {9}^5 {11}^6 {13}^6 {15}^3

{15}^3 {17}^6 {19}^6 {21}^5 {23}^4 {25}^3 {27}^2 {29}

]... every feature is perfectly reproduced by the Coxeter orbit of a8 , as listed in ...



... This is part of the 'hidden geometrical structrure' advertised in the title ... **projecting ...[roots]... down**

from R^r onto the two-dimensional eigenspace of w for the eigenvalue exp(2 pi i / h) , the relative angles become exactly the fusing angles ... the 'fusing angle' ... U^c_ab :

m_c^2 = m_a^2 + m_b^2 + 2 m_a m_b cos U^c_ab

... and **the line-lengths ... become ... exactly the masses** ...

The picture emerging is that each particle type should be associated with the orbit of the Coxeter element, and it turns out that the antiparticle is associated with the negative orbit ... ".

Bertram Kostant in his paper "On finite subgroups of SU(2), simple Lie algebras, and the McKay correspondence" (Proc. Natl. Acad. Sci. USA 81 (1984) 5275-5277) said:

"... the McKay correspondence ... sets up a bijection between the set of all isomorphism classes of subgroups ...[G of]... SU(2) and the set of all isomorphism classes of complex Lie algebras of type A, D, and E ... the correspondence ... has ... elegant ...[structure]... in terms of the orbits of the Coxeter element for the associated simple Lie algebra and that the numbers involved come in a beautiful way from the root structure ... the product decomposition ...

P_G(t) = SUM(i = 0 to h) z_i t^i / (1 - t^a) (1 - t^b) = z(t)_i / (1 - t^a) (1 - t^b)

... where z(t) = SUM(n = 0 to h) z_n t^n ...

... arises not from reflection group theory but from a study of the Coxeter element. As a consequence, the numbers involved are directly related to the root structure ... But then, if one proceeds to make the connection with the reflection group theory, one obtains as a theorem ... that the number of reflection hyperplanes is given in terms of the Coxeter number and the lesser degree of the two fundamental invariants is given in terms of the highest coefficient of the maximal root ...

Table 1. *a* and *b* values for the five types of subgroups of SU(2)

F^*	$\mathfrak{g}$	a	b	h
Z_n^*	A_{2n-1}	2	$2n$	$2n$
Δ_n^*	D_{n+2}	4	$2n$	$2n + 2$
A_4^*	E_6	6	8	12
S_4^*	E_7	8	12	18
A_5^*	E_8	12	20	30

If F is a subgroup of SO(3) we denote by F^* its inverse image in SU(2) with respect to the usual double covering SU(2) → SO(3). Δ_n, the dihedral group of order $2n$.

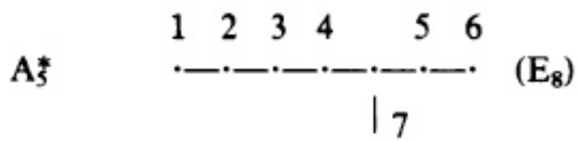
... One has a = 2d where d = d_i* is the coefficient of the highest root ... corresponding to the special node i* (... 6 for E8). Furthermore, b is given by b = h + 2 - a . In addition one has ... a b = 2 |G| ...[where |G| is the order of F*]...

Now let PHI be the set of roots phi of (h , g) such that (phi , psi) > 0 . One has

Card PHI = 2 h - 3

and one may refer to PHI as a Heisenberg system since a set of corresponding root vectors span a Heisenberg Lie subalgebra of g of dimension 2h - 3 . ...[for g = E8, 2h - 3 = 2x30 - 3 = 60 - 3 = 57]...

The values for the z(t)_i polynomials after labeling the nodes (eliminating i*) on the Dynkin diagram of ... E8 are ...

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z(t)_1 = t   + t^11 + t^19 + t^29

z(t)_2 = t^2 + t^10 + t^12 + t^18 + t^20 + t^28

z(t)_3 = t^3 + t^9  + t^11 + t^13 + t^17 + t^19 + t^21 + t^27

z(t)_4 = t^4 + t^8  + t^10 + t^12 + t^14 + t^16 + t^18 + t^20 + t^22 + t^26

z(t)_5 = t^6 + t^8  + t^12 + t^14 + t^16 + t^18 + t^22 + t^24

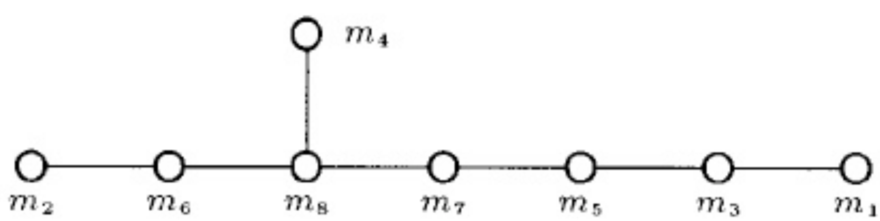
z(t)_6 = t^7 + t^13 + t^17 + t^23

z(t)_7 = t^6 + t^10 + t^14 + t^16 + t^20 + t^24
```

... proofs will appear in ... Kostant, B. (1984) On Finite Subgroups of SU(2), Simple Lie Algebras, and the McKay Correspondence in The Mathematical Heritage of Elie Cartan, ed. Dixmier, J. ...".

H. W. Braden, E. Corrigan, and P. E. Dorey (Nuclear Physics B338 (1990) 689-746) said:

"... The masses are attached to the Dynkin diagram as follows [similar to Grimm and Nienhuis but different from Kostant]:



... It is interesting to note that the characteristic polynomial of M2 factorises nicely into four quadratic factors in this case. Setting x = 1/2 m^2 we have the polynomial

```
(x^2 - (3/2)(5 - sqrt(5)) x +     3(5 - 2sqrt(5)))
(x^2 - (3/2)(5 + sqrt(5)) x +     3(5 + 2sqrt(5)))
(x^2 - (3/2)(5 - sqrt(5)) x + (3/2)(5 -  sqrt(5)))
(x^2 - (3/2)(5 + sqrt(5)) x + (3/2)(5 +  sqrt(5)))
```

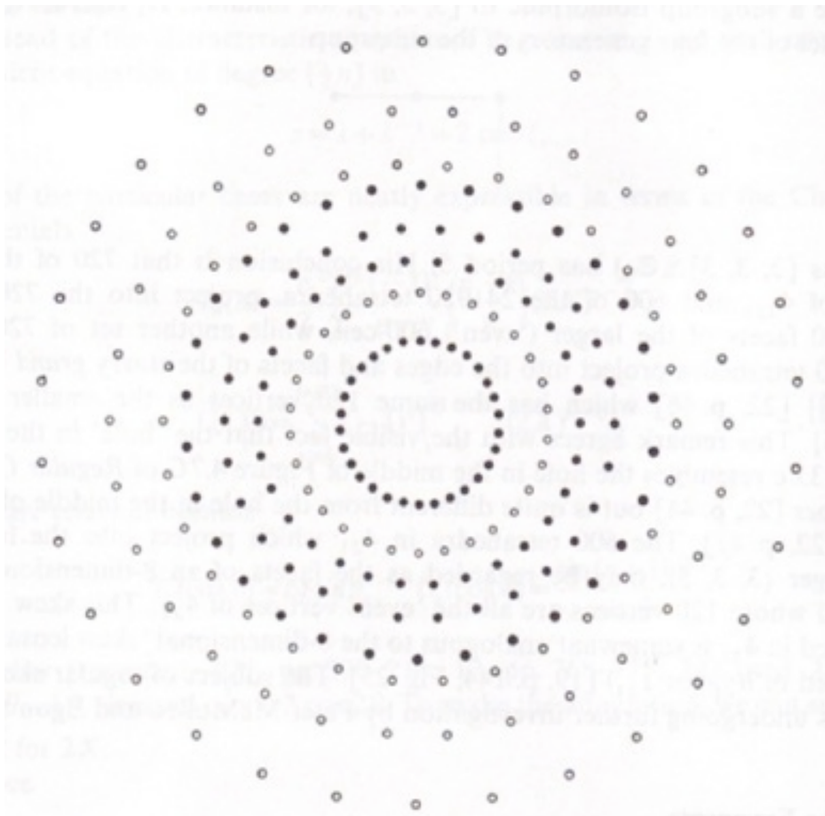
Again, the couplings are ...[here is a partial list, with somewhat altered notation]...

```
S11 =  1   11
S12 =  7   13
S13 =  2   10   12
S14 =  6   10   14
S15 =  3    9   11   13
S16 =  6    8   12   14
S17 =  4    8   10   12   14
S18 =  5    7    9   11   13   15

S88 =  1    3^2  5^3  7^4  9^5 11^6 13^6 15^3
```

... where ... the fusion angles are expressed in multiples of pi / 30 ... The magnitude of the couplings fits to the general rule with r = 8 and h = 30. ...".

The 240 root vectors of E8 split into 8 sets of 15 each. Although in 8-dimensional space all 240 root vectors are of the same length, **in this projection to 2-dimensional space each of the 8 sets of 15 has its own length** as seen in this projection to 2-dimensional space taken from "Regular and Semi-Regular Polytopes III"(Math. Zeit. 200 (1988) 3-45) by H. S. M. Coxeter, reprinted in "Kaleidoscopes: Selected Writings of H. S. M. Coxeter" (Wiley 1995)):



The 8 shells (8x15 = 240 root vectors) of E8 root vectors are shown above as black and white dots, with **the ratio of lengths of the 4 white-dot shells to the corresponding 4 black-dot shells being the Golden Ratio 1.618...** .

As Patrick Dorey (arXiv hep-th/9212143) said: "**... projecting ...[roots]... down from R^r onto the two-dimensional eigenspace** ... **the line-lengths ... become ... exactly the masses ...**", so the radii of the 8 shells shown above correspond to the 8 bound states of Zamolodchikov, with the following relative lengths:

1	oxoooooo	1
2	oxxooooo	1.618034...
3	oxxxoooo	1.989044...
4	oxxxxooo	2.404867...
5	oxxxxxoo	2.956295...
6	oxxxxxxo	3.218340...
7	oxxxxxxx	3.891157...
8	xxxxxxxx	4.783386...

2 - Why the molecular N-chain structure has natural E8 symmetry

Here is a physical way to see what is happening in the E8 quasiparticle experiment by Coldea et al (Science 327 (8 January 2010) 177-180):

Start with the Ising N-chain of binary states.

... oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo ...

The N-chain has a total of 2^N states which can be represented by the real Clifford algebra Cl(N).

> Note: For simplicity assume that N is divisible by 8 and 16 - if it were not, you could just go to the next highest number that is so divisible.

By 8-periodicity of real Clifford algebras, Cl(n) can be factored into the tensor product

Cl(N) = Cl(8) x ... (N/8) times tensor product ... x Cl(8)

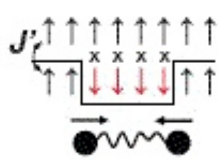
Therefore, the N-chain of binary elements can be represented as an (N/8)-chain of 8 binary elements each

... oooooooo oooooooo oooooooo oooooooo oooooooo oooooooo oooooooo oooooooo ...

with each 8-element link of the (N/8)-chain being representable by the 2^8 = 16x16 = 256-dimensional real Clifford algebra Cl(8)

and with the 8 binary elements of each 8-element link corresponding to the 8 Zamolodchikov kink-pair bound states

oxoooooo
oxxooooo
oxxxoooo
oxxxxooo
oxxxxxoo
oxxxxxxo
oxxxxxxx
xxxxxxxx

that were represented graphically by Coldea et al as 

Roughly speaking, there are 8 Zamolodchikov states because:

> an Ising chain of N binary elements naturally breaks up into a chain of links of 8 elements each; and
>
> for each 8-element chain there are only 8 possible different degrees of separation of a 2-kink bound state (including the 1-kink and 0-kink possibilities that are not really 2-kink but are 1 free kink and no kink).

As Coldea et al (Science 327 (8 January 2010) 177-180) said: "... Zamolodchikov proposed precisely eight "meson" bound states (the kinks playing the role of quarks), with energies in specific ratios given by a representation of the E8 exceptional Lie group ... At least five modes can be clearly observed ...", it appears that

the first 5 bound states

```
oxoooooo
oxxooooo
oxxxoooo
oxxxxooo
oxxxxxoo
```

are clearly observable, while the last 3 bound states

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oxxxxxxo
oxxxxxxx
xxxxxxxx
```

(including the 1-kink and 0-kink possibilities that are not really 2-kink but are 1 free kink and no kink) are substantially indistinguishable from unbound states.

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3 - How N-chain E8 symmetry leads to the E8 Physics model and the Origin of our Universe

The binary N-chains discussed above in parts 1 and 2 suggest experimental support for the E8 Physics model.

Each of the 256-dimensional Cl(8) of each of the (N/8) 8-element links of the binary N-chain has graded structure

1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1

which can, using triality and other mappings, represent a copy of 248-dimensional E8 whose graded structure is

8 + 28 + 56 + 64 + 56 + 28 + 8

with E8 physical interpretations as:

28 + 8 of 64 + 28 = 64 = 8x8 = 8 K-K spacetime Position dim each with 8 dual Momenta (Dirac Gammas)

28+28 of 64 = two copies of D4

- one D4 containing the Standard Model Gauge Groups

- the other D4 containing an SU(2,2) = Spin(2,4) Conformal Group for MacDowell-Mansouri Gravity

8+56 = 64 = 8x8 = 8 covariant components in 8-dim spacetime of 8 fundamental fermion particles

56+8 = 64 = 8x8 = 8 covariant components in 8-dim spacetime of 8 fundamental fermion antiparticles

The physical interpretations of the parts of E8 allow them to be combined to form a classical Lagrangian with Gravity and Standard Model Gauge terms and Fermion Particle/Antiparticle terms integrated over 8-dimensional Kaluza-Klein Spacetime, which in turn (see www.valdostamuseum.org/hamsmith/E8physicsbook.pdf) uses concepts of Mayer, Hua, Wyler, Batakis, Yamawaki, Segal, and others to produce realistic results including particle masses, force strengths, etc.

The E8 Lagrangian structure is inherently local and classical, but naturally forms a Global Algebraic Quantum Field Theory by by embedding E8 into Cl(8) and using a copy of Cl(8) to represent each Local Lagrangian Region, and then forming a Global Structure by taking the tensor products of the copies of Cl(8). Due to Real Clifford Algebra 8-periodicity, a sufficiently large tensor product of Cl(8) can represent any Real Clifford Algebra. Just as the completion of the union of all tensor products of 2x2 complex Clifford algebra matrices produces the usual Hyperfinite II1 von Neumann factor that describes creation and annihilation operators on the fermionic Fock space over C^(2n) (see John Baez's Week 175), we can take the completion of the union of all tensor products of Cl(8) to produce a real generalized Hyperfinite II1 von Neumann factor which will be our E8 Global Algebraic Quantum Field Theory.

The binary N-chains discussed above in parts 1 and 2 suggest experimental support for the E8 Physics model picture of the Cosmological Origin of our Universe, in which our universe is formed beginning with a tensor product chain of local E8 structures (each embedded into Cl(8) real Clifford algebras, and each denoted in the following figure by o, with ... and ... denoting contnuation)

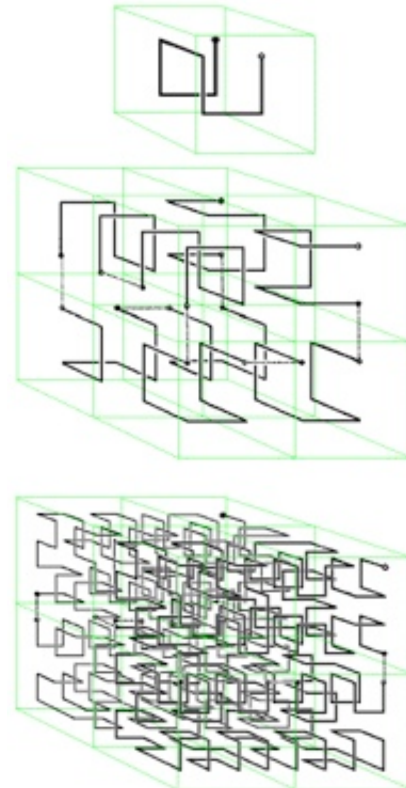
... oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo ...

That linear pregeometry, like a long line of yarn, then folds or weaves into a higher-dimensional array or tapestry of Cl(8) that produces something like a Moore space-filling curve. Acccording to a web page of V. B. Balayoghan:

> "... The Hilbert and Moore curves use square cells -- the level n curve has 4^n cells (and hence 4^n - 1 lines). The Moore curve has the same recursive structure as the Hilbert curve, but ends one cell away from where it started. The Hilbert curve starts and ends at opposite ends of a side of the unit square. ...".

According to a web page by William Gilbert:

> "... We exhibit a direct generalization of Hilbert's curve that fills a cube. The first three iterates of this curve are shown.



In constructing one iterate from the previous one, note that the direction of the curve determines the orientation of the smaller cubes inside the larger one.

The initial stage of this three dimensional curve can be considered as coming from the 3-bit reflected Gray code which traverses the 3-digit binary strings in such a way that each string differs from its predecessor in a single position by the addition or subtraction of 1. The kth iterate could be considered a a generalized Gray code on the Cartesian product set {0,1,2,...,2^k-1}^3.

The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube. ...".

According to Numerical Recipes in C, by Press, Teukolsky, Vettering, and Flannery (2nd ed, Cambridge 1992):

"... A Gray code is a function G(i) of the integers i, that for each integer N > 0 is one-to-one for 0 < i < 2^N -1, and that has the following remarkable property: The binary representation of G(i)and G(i+1) differ in exactly one bit. an example of a Gray code ... is the sequence ...[

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0000 ( 0=0000), 0001 ( 1=0001), 0011 ( 2=0010), 0010 ( 3=0011),
0110 ( 4=0100), 0111 ( 5=0101), 0101 ( 6=0110), 0100 ( 7=0111),
1100 ( 8=1000), 1101 ( 9=1001), 1111 (10=1010), 1110 (11=1011),
1010 (12=1100), 1011 (13=1101), 1001 (14=1110), 1000 (15=1111)
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]... for i = 0, ... 15. The algorithm for generating this code is simply to form ... XOR of i with 1/2 (integer part). ... G(i) and G(1+1) differ in the bit position of the rightmosst zero bit of i ... Gray codes can be useful when you need to do some task that depends intimately upon the bits

> of i, looping over many values of i. Then, if there are economies in repeating the task for values differing only by one bit, it makes sense to do things in Gray code order rather than consecutive order. ...".

Calderbank, Rains, Shor, and Sloane (arXiv \quant-ph/9608006) show that whereas many useful classical-error-correcting codes are binary, over the Galois field GF(2) = {0,1}, quantum-error-correcting codes are quaternary, over the Galois field GF(4) = {0,1,w,w^2} where w = (1/2)(- 1 + sqrt(3) i) and w^2 = (1/2)(- 1 - sqrt(3) i).

Hammons, Kumar, Calderbank, Sloane, and Sole (arXiv math.CO/0207208) say:

> "...Some famous nonlinear codes found by Nordstrom-Robinson, Kerdock, Preparata, Goethals and others, more powerful than any linear codes ... are ideals in polynomial rings over the ring of integers mod 4. ... the Gray map translates a quaternary code with high minimal Lee or Euclidean distance into a binary code of twice the length with high minimal Hamming distance. ... The Kerdock and Preparata codes of length 16 coincide, giving the Nordstrom-Robinson code. This is the unique binary code of length 16, minimal distance 6, containing 256 words. In this case K ... the cyclic code of length 16 over Z4 ...[plus]... adjoining a zeo-sum check symbol. ... is the ... octacode ... the unique self-dual quaternary code of length 8 and minimal Lee weight 6, or ... the 'glue code' required to construct the 24-dimensional Leech lattice from eight copies of the face-centered cubic lattice. ... The Nordstrom-Robinson code is the binary image of the octacode under the Gray map. ...".

The Gray code structure of our N-chain Cl(8) Universe may give us a quantum logic structure that could realize Paola Zizzi's view of the Inflation Phase of our Universe as a Quantum Computer expressed in arXiv gr-qc/0007006 and gr-qc/0103002 where she says:

> "... We consider a quantum gravity register that is a particular quantum memory register which grows with time, and whose qubits are pixels of area of quantum de Sitter horizons. At each time step, the vacuum state of this quantum register grows because of the uncertainty in quantum information induced by the vacuum quantum fluctuations. The resulting virtual states, (responsible for the speed up of growth, i.e., inflation), are operated on by quantum logic gates and transformed into qubits. ... We also show that the bound on the speed of computation, the bound on clock precision, and the holographic bound, are saturated by the ... quantum growing network ... QGN. ... The model of ... QGN ... described here is exactly solvable, and ... could even undergo conscious experiences, if we believe ... that conscious experiences are due to decoherence of tubulins-qubits ...
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> ... One could argue that the QGN discussed in this paper, is one of the attractors of some self-organizing system. That self-organizing system might be some kind of non local and non causal space-time structure made up of entangled qubits ... Although that system does not represent any physical space-time, it can be considered as a proto space-time, which is the seed of physical quantum space-time. ... we think that ... a random and non-local structure exists just below the Planck scale. At the Planck scale, the random network has already self-organized into

the QGN. Indeed, we believe that the quantum beginning of physical space-time took place at the node 0 (the Hadamard gate) of the QGN. Quantized time appears as the result of the transformation of virtual states (vacuum energy) into qubits (quantum information) at the nodes of a quantum network. ...

... The speed up of the growth of the QGN, is due to virtual states and it is responsible for quantum inflation. If virtual states were absent in the quantum network, the growth would be much slower. In that case, the early universe could be interpreted as a 2^n lattice (n=0,1,2…), represented by the regular tree graph ...

... we have three "degrees" (or phases) for the beginning of the universe:

- The (perhaps eternal) presence of the proto space-time below the Planck scale.
- The beginning of (quantum) inflation at the Planck time.
- The end of inflation and beginning of existence.

... during inflation, the universe can be described as a superposed state of quantum ... The self-reduction of the superposed quantum state is ... reached at the end of inflation ...

... The beginning of existence of the universe (at the end of inflation due to decoherence) coincided with a cosmic conscious event ... of which our brain structure is still reminiscent. ...".

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