E8 Real Forms and Evolution of our Universe

Frank Dodd (Tony) Smith, Jr. - 2017

Thanks to Alessio Marrani for pointing out some errors and lack of clarity in my work up to now on my E8 Physics model (viXra 1602.0319 etc) regarding E8 Real Forms.
He motivated me to try to think more clearly about E8 Real Forms and their physical significance, resulting in this paper.
Any errors herein are mine but any good basic ideas are due to Alessio.

> Errors and lack of clarity in my work up to now with respect to E8 Real Forms are somewhat similar to what happened earlier with my use of the terms Spinor and Pinor, something that I attempted to clarify on my web site at http://valdostamuseum.com/hamsmith/clfpq.html#SPIN where I said "... Sometimes I use the term Spinor, or Spin(p,q), when I really should use the term Pin, or Pin(p,q). A physical significance of the difference is that Spinors and Spin(n) are related to the even subalgebra of the Clifford algebra Cl(p,q) (where Cle(p,q) = Cl(p,q-1) and Cle(p,0) = Cl(0,p-1))) and so do not contain some reflection-related characteristics (such as parity reversal, etc.), while such things are contained in Pin and Pin(p,q) because they are related to the full Clifford algebra Cl(p,q) including its odd part. A paper by Marcus Berg, Cecile DeWitt-Morette, Shangjr Gwo, and Eric Kramer, math-ph/0012006, discusses Pin and Spin. ... I hope that readers can see what I mean from context, because I have misused the terminology in so many places throughout my materials that I have not had the energy to correct them. However, I do not think that my misuse of math terminology has resulted in wrong physics. That is, the ... E8 ... Physics model is in my opinion physically realistic and valid, even though my description of it may use some incorrect math terminology. ...". The same hope is applicable here.

Wikipedia says "... There is a unique complex Lie algebra of type E8, corresponding to a complex group of complex dimension 248. ... This is simply connected, has maximal compact subgroup the compact form ... of E8, and has an outer automorphism group of order 2 generated by complex conjugation. As well as the complex Lie group of type E8, **there are three real forms of the ... E8 ... Lie algebra** ...
compact form E8(-248) [and] **split form, EVIII (or E8(8))** [and] **EIX (or E8(-24))** ...".

In E8 Physics:

The Compact Form E8(-248) with Symmetric Space E8 / SO(16) represents Our Planck Scale Universe when it emerged from its Parent Universe by Quantum Fluctuation.

The Split Form EVIII E8(8) with Symmetric Space E8 / SO(8,8) represents Our Universe during Octonionic Inflation with Non-Unitary Quantum Processes.

The form EIX E8(-24) with Symmetric Space E8 / SO*(16) represents Our Universe with Quaternionic Unitary Quantum Processes after the end of Inflation.

Wikipedia says: “...

The **compact form ... E8(-248**) ... is simply connected and has trivial outer automorphism group ... maximal subgroups of E8 ...[include]... E7 x Sl(2)/(-1,-1) and E6 x SU(3)/(Z/3Z) ... Symmetric space ...
E8 / SO(16) ...
... E8 / E7 x Sp(1) ...

The **split form, EVIII (or E8(8))** ... has maximal compact subgroup Spin(16)/(Z/2Z), fundamental group of order 2 (implying that it has a double cover, which is a simply connected Lie real group but is not algebraic ...) and has trivial outer automorphism group ... Symmetric space ...
E8(8) / SO(16) ... E8(8) / SO(8,8) ... E8(8) / Sk(8,H) [= E8(8) / SO*(16)]...
... E8(8) / E7(7) x SL(2,R) ... E8(8) / E8(8) / E7(-5) x SU(2) ...

EIX (or E8(-24)) ... has maximal compact subgroup E7×SU(2)/(−1,−1), fundamental group of order 2 (again implying a double cover, which is not algebraic) and has trivial outer automorphism group ... Symmetric Space ...
E8(-24) / SO(12,4) ... E8(-24) / Sk(8,H) [= E8(-24) / SO*(16)]...
... E8(-24) / E7(-5) x SU(2) [Quaternion-Kahler] ... E8(-24) / E7(-25) x SL(2,R) ...”.

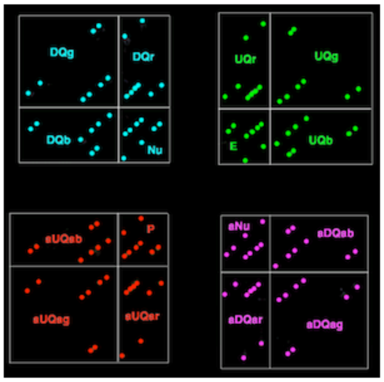
According to the book Einstein Manifolds by pseudonymous Arthur L. Besse (a group of French mathematicians including Marcel Berger):

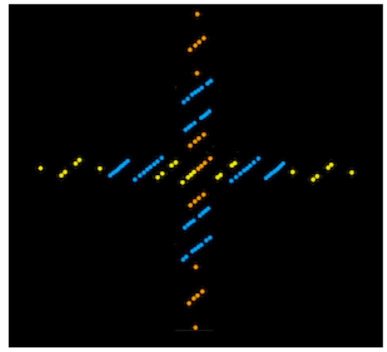
Compact E8(-248) has symmetric space E8 / SO(16) of dimension 128 and rank 8 with Isotropy representation Spin(16) that is **Rosenfeld’s elliptic projective plane** (Ca x Ca)P2 (where Ca = Cayley Octonions and x = tensor product)

Split EVIII E8(8) has symmetric space E8 / SO(16) of dimension 128 and rank 8 with Isotropy representation Spin(16) that is **Rosenfeld’s hyperbolic projective plane** (Ca x Ca)P2hyp

EIX E8(-24) has symmetric space E8 / E7 x SU(2) of dimension 112 and rank 4 with Isotropy representation /\2 E7 x SU(2) that is the Set of the (H x Ca)P2hyp ‘s in “(Ca x Ca)P2hyp”
(where /\ denotes exterior product representation and H = Quaternions)

Since my E8 Physics model is based on the 240 E8 Root Vectors being decomposed into 128 corresponding to D8 Half-Spinors and 112 corresponding to D8 Root Vectors





the Real Forms of E8 for my E8 Physics model with Octonionic 8-dim Spacetime prior to Post-Inflation Transition to (4+4)-dim Quaternionic M4 x CP2 Kaluza-Klein are

Compact E8(-248) with Rosenfeld's elliptic projective plane
and
NonCompact Split EVIII E8(8) with Rosenfeld's hyperbolic projective plane

Robert Gilmore in Phys. Rev. Lett. 28 (1972) 462-464 showed that Armand Wyler's use to calculate force strengths and particle masses as ratios of volumes of compact domains of unit radius instead of measures on noncompact projectively related domains is juatified, saying "... the replacement of a divergent value by a finite value can lead to a well-defined and significant result. The occurrence of the Euclidean volumes V(Qn) and V(Dn) should be considered a strong point of Wyler's result, rather than an objectionable feature. These volumes arise naturally as the normalizing coefficients in the Poisson and Bergman kernels, which are reproducing functions and are defined in a nonlinear way. The Poisson kernel is the image of a space-time scalar Green's function, when both arguments of the kernel are on the boundary Qn of Dn ... Wyler's work has pointed out that it is possible to map an unbounded physical domain - the interior of the forward light cone - onto the interior of a bounded domain on which there also exists a complex structure. This mapping should prove of immense calculational value in the future. This transformation from unbounded to bounded complex domains is mathematically rigorous, and is valid ...".

My E8 Physics model makes use of Wyler's valid technique in calculation of ratios of force strengths and particle masses, and it seems clear to me that the validity extends to use of both Compact E8(-248) and NonCompact Split EVIII E8(8) Real Forms in the basic structure of E8 Physics, so

Compact E8(-248) and **NonCompact Split EVIII E8(8)**
are both useful in describing E8 Physics of 8-dim Octonionic Spacetime
at High (from Planck through Inflation to End of Inflation) Energies.
EIX E8(-24) is not useful in that Energy Range
because it does not have a Symmetric Space with symmetry SO(16) or SO(8,8)

After the End of Inflation E8 Physics has a transition
from 8-dim Octonionic Spacetime to
(4+4)-dim Quaternionic Kaluza-Klein Spacetime M4 x CP2
where M4 is 4-dim physical Minkowski Spacetime
and CP2 = SU(3) / SU(2) x U(1) Internal Symmetry Space
so that
the **Symmetric Space of E8 Physics** goes from Octonionic SO(16) or SO(8,8) to
Quaternionic Sk(8,H) = SO*(16)
therefore Compact E8(-248), with no SO*(16) symmetry, is no longer useful
and
the useful **Real Forms of E8 for E8 Physics after Inflation** are
NonCompact Split EVIII E8(8) with **E8(8) / Sk(8,H) = E8(8) / SO*(16)**
and
EIX E8(-24) with **E8(-24) / Sk(8,H) = E8(-24) / SO*(16)**

How does the transition from SO(16) and SO(8,8) to SO*(16) work ?

Sigurdur Helgason,
in his 1978 book Differential Geometry, Lie Groups, and Symmetric Spaces, says: "...

$\boldsymbol{SO(n, C)}$: The group of matrices g in $\boldsymbol{SL(n, C)}$ which leave invariant the quadratic form

$$z_1^2 + ... + z_n^2, \qquad \text{i.e.,} \quad {}^t gg = I_n.$$

$\boldsymbol{SO(p, q)}$: The group of matrices g in $\boldsymbol{SL(p+q, R)}$ which leave invariant the quadratic form

$$-x_1^2 - ... - x_p^2 + x_{p+1}^2 + ... + x_{p+q}^2, \qquad \text{i.e.,} \quad {}^t g I_{p,q} g = I_{p,q}.$$

We put $\boldsymbol{SO(n)} = \boldsymbol{SO(0, n)} = \boldsymbol{SO(n, 0)}$.

$\boldsymbol{SO^*(2n)}$: The group of matrices in $\boldsymbol{SO(2n, C)}$ which leave invariant the skew Hermitian form

$$-z_1\bar{z}_{n+1} + z_{n+1}\bar{z}_1 - z_2\bar{z}_{n+2} + z_{n+2}\bar{z}_2 - ... - z_n\bar{z}_{2n} + z_{2n}\bar{z}_n.$$

Thus $g \in \boldsymbol{SO^*(2n)} \Leftrightarrow {}^t g J_n \bar{g} = J_n, \ {}^t gg = I_{2n}$.

... The groups listed above are all topological Lie subgroups of a general linar group ...

The Lie algebra for each of the groups above ... will be denoted by ... small ... letters ...

$\mathfrak{so}(n, C)$: {all $n \times n$ skew symmetric complex matrices},

$\mathfrak{so}(p, q)$: $\left\{\begin{pmatrix} X_1 & X_2 \\ {}^tX_2 & X_3 \end{pmatrix} \middle| \begin{array}{l} \text{All } X_i \text{ real, } X_1, X_3 \text{ skew symmetric of order} \\ p \text{ and } q\text{, respectively, } X_2 \text{ arbitrary} \end{array}\right\}$,

$\mathfrak{so}^*(2n)$: $\left\{\begin{pmatrix} Z_1 & Z_2 \\ -\bar{Z}_2 & \bar{Z}_1 \end{pmatrix} \middle| \begin{array}{l} Z_1, Z_2 \; n \times n \text{ complex matrices,} \\ Z_1 \text{ skew, } Z_2 \text{ Hermitian} \end{array}\right\}$,

$$SO^*(2n) \cap U(2n) = SO(2n) \cap Sp(n, C) = SO(2n) \cap Sp(n) \approx U(n).$$

Robert Gilmore
in his 1974 book Lie Groups, Lie Algebras, and Some of Their Applications, says "...

WEYL UNITARY TRICK. A real space with a signature (N_+, N_-) can be converted to a space with metric $(N_+ + N_-, 0)$ by choosing a new set of bases

$$(\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_{N_+}, \mathbf{e}_{N_++1}, \ldots, \mathbf{e}_{N_++N_-}) \to (\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_{N_+}, i\mathbf{e}_{N_++1}, \ldots, i\mathbf{e}_{N_++N_-}) \qquad (3.19)$$

Of course, we have to go outside the field of real numbers to perform this transformation. For example, the space–time of special relativity has metric $(+++-)$ with respect to the real contravariant bases (x, y, z, ct) but metric $(++++)$ with respect to (x, y, z, ict). This transformation from a mixed to a positive metric is called the **Weyl unitary trick**. It was apparently first used by Minkowski.

Orthogonal	$SO(n, c)$	(M)	
	$SO(p, q; r)$	$\left[\begin{array}{c\|c} A & B \\ \hline B^t & C \end{array}\right]$	$\left[\begin{array}{c\|c} A & \\ \hline & C \end{array}\right]$
	$SO^*(2n)$	$\left[\begin{array}{c\|c} A & B \\ \hline -B^* & A^* \end{array}\right]$	B A iB
	$SO(n, r)$	(A)	

3. ORTHOGONAL GROUPS. The real and complex orthogonal groups $SO(n, r)$ and $SO(n, c)$ preserve the canonical bilinear symmetric metric $g_{ij} = \delta_{ij}$. By (1.5), their algebras consist of real and complex antisymmetric matrices:

$$M = A^{ij} E_{ij}^{(n)}$$
$$M^t = -M \qquad A^{ij} = -A^{ji} \tag{1.20}$$

The complex extension of $SO(n, r)$ is $SO(n, c)$.

The Lie algebra for $SO(p, q; r)$ is related to the Lie algebra for $SO(p + q; r)$ by the Weyl unitary trick:

$$\begin{array}{cc} & \begin{array}{cc} \longleftarrow p \longrightarrow & \longleftarrow q \rightarrow \end{array} \\ \begin{array}{c} \uparrow \\ p \\ \downarrow \\ \uparrow \\ q \\ \downarrow \end{array} & \left[\begin{array}{c|c} M_{11} & M_{12} \\ \hline -M_{12}{}^t & M_{22} \end{array}\right] \end{array} \; \begin{array}{c} \text{Weyl} \\ \text{unitary} \\ \rightarrow \\ \text{trick} \end{array} \; \begin{array}{cc} & \begin{array}{cc} \longleftarrow p \longrightarrow & \longleftarrow q \rightarrow \end{array} \\ \begin{array}{c} \uparrow \\ p \\ \downarrow \\ \uparrow \\ q \\ \downarrow \end{array} & \left[\begin{array}{c|c} M_{11} & iM_{12} \\ \hline -iM_{12}{}^t & M_{22} \end{array}\right] \end{array}$$

$$\mathfrak{so}(p + q; r) \qquad\qquad\qquad \mathfrak{so}(p, q; r) \tag{1.21}$$

Here M_{11}, M_{22} are the compact subalgebras for $SO(p, r)$ and $SO(q, r)$. The subspace generated by the $p \times q$ matrices M_{12} and iM_{12} are compact generators for $SO(p + q; r)$ and noncompact generators for $SO(p, q; r)$, respectively.

The matrix elements of M_{12} are all real. Under complex extension,

$$SO(p, q; r) \xrightarrow[\text{extension}]{\text{complex}} SO(p, q; c) \tag{1.22}$$

It is easily verified that $SO(p + q;\, c)$ and $SO(p,\, q;\, c)$ have identical Lie algebras. Therefore, the groups $SO(p,\, q;\, r)$ are all real forms of the group $SO(p + q;\, c)$.

The group $SO^*(2n)$ is the subgroup of $SO(2n,\, c)$ which preserves the sesquilinear antisymmetric metric.[1] With respect to the antisymmetric metric

$$\left[\begin{array}{c|c} & I_n \\ \hline -I_n & \end{array}\right] \tag{1.23}$$

it is easily verified that the Lie algebra of $2n \times 2n$ matrices for $SO^*(2n)$ has the structure

$$\left[\begin{array}{c|c} M_{11} & M_{12} \\ \hline -M_{12}^* & M_{11}^* \end{array}\right] \qquad \begin{array}{ll} M_{11} & \text{skew symmetric} \\ M_{12} & \text{hermitian} \end{array} \tag{1.24}$$

It may be easily verified that the complex extension of this algebra is identical to the algebra of $SO(2n,\, c)$.

A convenient set of bases for the common complex extension is

$$O_{ij}^{(n)} = E_{ij}^{(n)} - E_{ji}^{(n)} = -O_{ji}^{(n)} \tag{1.25}$$

The commutation relations for the generators $O_{ij}^{(n)}$ are given by

$$[O_{ij}^{(n)}, O_{rs}^{(n)}] = O_{is}^{(n)}\, \delta_{jr} + O_{jr}^{(n)}\, \delta_{is} - O_{ir}^{(n)}\, \delta_{js} - O_{js}^{(n)}\, \delta_{ir} \tag{1.26}$$

6. ORIGIN OF THE EMBEDDING GROUPS $SO^*(2n)$ AND $SU^*(2n)$. The existence of the two unfamiliar "embedding groups" $SO^*(2n)$ and $SU^*(2n)$ as real forms of $SO(2n, c)$ and $Sl(2n, c)$ often comes as a rude shock to aficionados of Lie group theory. The difficulty is further compounded by the lack of a simple explanation for their existence. We present one now.

The group $U(n, c)$ consists of those $n \times n$ complex matrices which preserve the canonical positive definite sesquilinear symmetric metric $g_{ij} = \delta_{ij}$. Each matrix element is a complex number; the Lie algebra obeys

$$M_i{}^j = -M_j{}^{i*}$$

There is a canonical representation of the complex numbers by real-valued 2×2 matrices [see Chapter 1, (3.10)]

$$re^{i\phi} \to r\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

$$\begin{tikzcd} x + iy \arrow[d] & (x + iy)^* \arrow[d] \\ \begin{bmatrix} x & y \\ -y & x \end{bmatrix} & \begin{bmatrix} x & -y \\ y & x \end{bmatrix} \end{tikzcd} \tag{1.38}$$

Under this representation of the complex numbers, every complex entry in $U(n, c)$ is replaced by a real 2×2 matrix. Since the $2n \times 2n$ *real* matrices so obtained preserve the metric

$$g'_{ij} = \delta_{ij} \otimes I_2 \tag{1.39c}$$

they form a subgroup of $SO(2n, r)$.

We investigate the Lie algebra of this subgroup under

$$u_i{}^j \to r_i{}^j + ic_i{}^j \tag{1.40}$$

$$\begin{array}{rr l}
\text{in } C_n & & \begin{array}{cccccc} \quad & 1 & & 2 & & \end{array} \\
 & \text{in } R_{2n} & \begin{array}{ccccc} 1 & -1 & 2 & -2 & \end{array} \\
1 & \begin{array}{r} 1 \\ -1 \end{array} & \\
\delta M \to 2 & \begin{array}{r} 2 \\ -2 \end{array} & \\
 & &
\end{array}
\left[\begin{array}{cc|cc|c}
0 & c_1^1 & r_1^2 & c_1^2 & \\
 & & & & \cdots \\
-c_1^1 & 0 & -c_1^2 & r_1^2 & \\
\hline
-r_1^2 & c_1^2 & 0 & c_2^2 & \\
 & & & & \cdots \\
-c_1^2 & -r_1^2 & -c_2^2 & 0 & \\
\hline
\vdots & & \vdots & & \ddots
\end{array}\right] \tag{1.41}$$

Since the Lie algebra consists of real antisymmetric matrices, it is clearly a Lie subalgebra of $\mathfrak{so}(2n, r)$.

It is useful at this point to rearrange the rows and columns of this matrix:

$$\delta M \to \begin{array}{r l}
 & \begin{array}{cccccccc} 1 & 2 & \cdots & & -1 & -2 & \cdots & \end{array} \\
\begin{array}{r} 1 \\ 2 \\ \vdots \\ -1 \\ -2 \\ \vdots \end{array} &
\left[\begin{array}{cccc|cccc}
0 & r_1{}^2 & \cdots & & c_1{}^1 & c_1{}^2 & \cdots & \\
-r_1{}^2 & 0 & \cdots & & c_1{}^2 & c_2{}^2 & \cdots & \\
\vdots & \vdots & & & \vdots & \vdots & & \\
\hline
-c_1{}^1 & -c_1{}^2 & \cdots & & 0 & r_1{}^2 & \cdots & \\
-c_1{}^2 & -c_2{}^2 & \cdots & & -r_1{}^2 & 0 & \cdots & \\
\vdots & \vdots & & \ddots & \vdots & \vdots & & \ddots
\end{array}\right]
\end{array} \tag{1.42}$$

$$= \left[\begin{array}{c|c} A & B \\ \hline -B & A \end{array}\right] \qquad \begin{array}{ll} B^t = +B & \\ A^t = -A & \\ A, B & \text{real} \end{array} \tag{1.43}$$

The Lie algebra $\mathfrak{so}(2n, r)$ can be written as the direct sum of two vector subspaces $\mathfrak{k} \oplus \mathfrak{p}$:

$\mathfrak{k}$: The subspace of matrices (1.43), which form a $2n \times 2n$ real matrix representation of $\mathfrak{u}(n, c)$.

$\mathfrak{p}$: An orthogonal complementary subspace whose matrices have the general structure

$$\left[\begin{array}{c|c} C & D \\ \hline +D & -C \end{array}\right] \qquad \begin{array}{l} C^t = -C \\ D^t = -D \end{array} \tag{1.44}$$

In short, we have the decomposition

$$\begin{aligned} \mathfrak{so}(2n, r) &= \quad \mathfrak{u}(n, c) \quad \oplus \quad V_\perp \\ &= \left[\begin{array}{c|c} A & B \\ \hline -B & A \end{array}\right] \oplus \left[\begin{array}{c|c} C & D \\ \hline D & -C \end{array}\right] \\ &= \qquad \mathfrak{k} \qquad \oplus \qquad \mathfrak{p} \end{aligned} \tag{1.45}$$

It may be verified by direct calculation that the subspaces $\mathfrak{k}$, $\mathfrak{p}$ obey the commutation relations given symbolically by

$$\begin{aligned} [\mathfrak{k}, \mathfrak{k}] &\subset \mathfrak{k} \\ [\mathfrak{k}, \mathfrak{p}] &= \mathfrak{p} \\ [\mathfrak{p}, \mathfrak{p}] &= \mathfrak{k} \end{aligned} \tag{1.46}$$

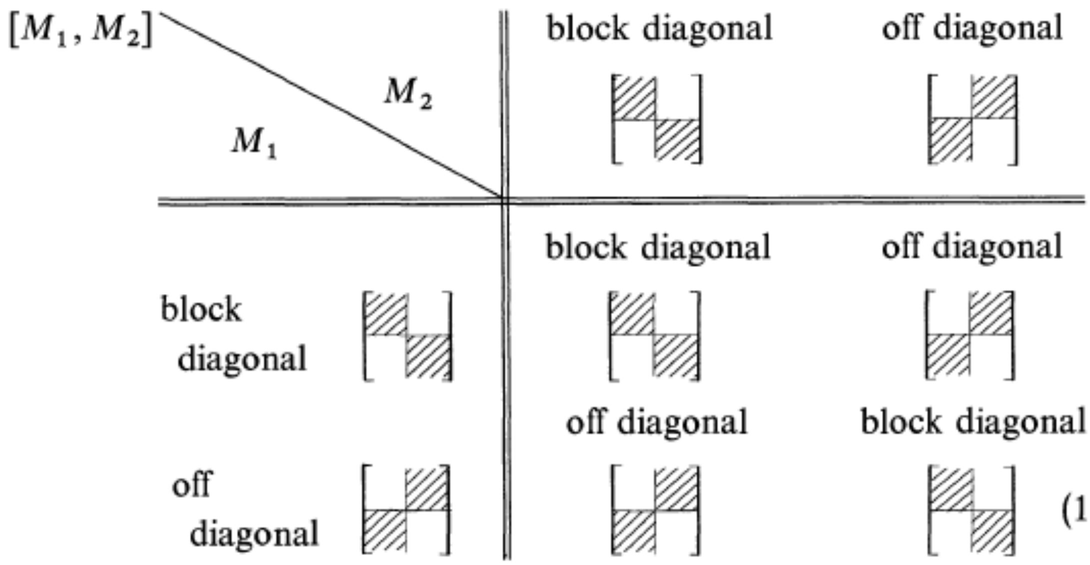
The commutation properties are most easily seen after making a similarity transformation using

$$S = \frac{1}{\sqrt{2}} \left[\begin{array}{c|c} +1I_n & -iI_n \\ \hline -iI_n & +1I_n \end{array}\right] \tag{1.47}$$

$$S \left[\begin{array}{c|c} A & B \\ \hline -B & A \end{array}\right] S^{-1} = \left[\begin{array}{c|c} A + iB & 0 \\ \hline 0 & A - iB \end{array}\right] \qquad \begin{array}{l} A^t = -A \\ B^t = +B \end{array} \tag{1.48k}$$

$$S \left[\begin{array}{c|c} C & D \\ \hline D & -C \end{array}\right] S^{-1} = \left[\begin{array}{c|c} 0 & D + iC \\ \hline D - iC & 0 \end{array}\right] \qquad \begin{array}{l} C^t = -C \\ D^t = -D \end{array} \tag{1.48p}$$

In this representation, the commutation properties of the matrix vector subspaces are especially easy to compute. The results are indicated in the following diagram:



(1.49)

Since $\mathfrak{so}(2n, r)$ is closed under commutation, any block diagonal submatrix arising from commutators belongs to the subalgebra $\mathfrak{k}$; any off-diagonal submatrix arising from commutators belongs to the subspace $\mathfrak{p}$.

If the Weyl unitary trick is now applied to the compact generators in the subspace $\mathfrak{p}$, they are converted to noncompact generators. Using (1.46), we find that the commutation relations obeyed by $\mathfrak{k}$ and $\mathfrak{p}^* = i\mathfrak{p}$ are

$$\begin{aligned} [\mathfrak{k}, \mathfrak{k}] &\subset \mathfrak{k} \\ [\mathfrak{k}, \mathfrak{p}^*] &= \mathfrak{p}^* \\ [\mathfrak{p}^*, \mathfrak{p}^*] &= (-)\mathfrak{k} \end{aligned} \tag{1.46*}$$

Therefore, the matrices $\mathfrak{k} \oplus \mathfrak{p}^*$ are closed under commutation and form the Lie algebra of some noncompact group

$$\begin{aligned} \mathfrak{so}^*(2n) &= \quad \mathfrak{k} \quad \oplus i\mathfrak{p} \\ &= \mathfrak{u}(n, c) \oplus i[\mathfrak{so}(2n, r) \bmod \mathfrak{u}(n, c)] \end{aligned} \tag{1.50}$$

Comment 1. The subalgebra of matrices in $\mathfrak{k}$ is antihermitian and therefore maps onto a compact group under the EXPonential mapping. The matrices in the subspace $\mathfrak{p}(\mathfrak{p}^* = i\mathfrak{p})$ are antihermitian (hermitian) and therefore map onto compact (noncompact) cosets. The maximal compact subgroup of $SO^*(2n)$ is $U(n, c)$.

Comment 2. The Lie algebra of $SO^*(2n)$ satisfies the condition (1.24). The group thus obeys the condition giving rise to (1.24) and may be defined accordingly: $SO^*(2n)$ is the subgroup of $SO(2n, c)$ which preserves the sesquilinear antisymmetric metric.

The algebra $\mathfrak{g}^*$, related to $\mathfrak{g}$ by the Weyl unitary trick

$$\mathfrak{g}^* = \mathfrak{k} \oplus i\mathfrak{p} \tag{1.63}$$

describes the embedding groups $SU^*(2n)$, $SO^*(2n)$, respectively. These results are summarized in Table 6.2.

TABLE 6.2

SUMMARY OF THE REAL FORMS OF THE CLASSICAL GROUPS

Process	Noncompact Group G^*	Maximal Compact Subgroup K	Associated Compact Group G	Dimension Coset Spaces G^*/K, G/K
Indefinite metric (p, q) preserving groups	$SO(p, q)$	$SO(p) \otimes SO(q)$	$SO(p+q)$	pq
	$SU(p, q)$	$S[U(p) \otimes U(q)]$	$SU(p+q)$	$2pq$
	$USp(2p, 2q) \simeq U(p, q; Q)$	$USp(2p) \otimes USp(2q) \simeq U(p; Q) \otimes U(q; Q)$	$USp(2p+2q) \simeq U(p+q; Q)$	$4pq$
Subfield restriction	$Sl(n, r)$	$SO(n)$	$SU(n, c)$	$\frac{n(n+1)}{2} - 1$
	$Sp(2n, r)$	$U(n, c)$	$USp(2n) \simeq U(n; Q)$	$n(n+1)$
Embedding groups	$SO^*(2n)$	$U(n, c)$	$SO(2n)$	$n(n-1)$
	$SU^*(2n)$	$USp(2n) \simeq U(n, Q)$	$SU(2n)$	$\frac{2n(2n-1)}{2} - 1$

.

..".

In his 2008 book Lie Groups, Physics, and Geometry, Robert Gilmore refers to his 1974 book containing the above quotes, saying: "... Many years ago I wrote the book Lie Groups, Lie Algebras, and Some of Their Applications (NewYork:Wiley, 1974). That was a big book: long and difficult. ... I ... promise[d] that some day I would ... rewrite and shrink the book ... in a way that was easy for students to acquire and to assimilate ...". However, I think that the details in the "long and difficult" book are important for working out details of physics models such as my E8 Physics, so I very much like it despite its being "difficult". For example beyond Real Forms of E8, the "long and difficult" book contains at page 349

$SU(2, 2)$	$+1$	$SO(4, 2)$	$SO(4, 2)$ is one of the groups whose Green's functions may give information on the fine structure constant.	13,14

13. A. Wyler, L'espace symétrique du groupe des équations de Maxwell, *C.R. Acad. Sci. Paris* **269**, Ser. A, 743–745 (1969).
14. A. Wyler, Les groupes des potentiels de Coulomb et de Yukawa, *C.R. Acad. Sci. Paris* **272**, Ser. A, 186–188 (1971).

which is perhaps the only serious reference in any influential math or physics books to the techniques of Armand Wyler that I use in my E8 Physics.

In summary,
here is how Our Universe Evolved in terms of my E8 Physics model:

When Our Planck Scale Universe emerged from its Parent Universe by Quantum Fluictuation it was described by SO(16) symmetry of Compact E8(-248).

When Our Universe was expanding rapidly during Octonionic Non-Unitary Inflation it unfolded from Finite Elliptic Compact to Infinite Hyperbolic NonCompact SO(8,8) symmetry of NonCompact Split EVIII E8(8).
That transition was a shifting of SO(16) symmetry from E8(-248) to E8(8) followed by a Weyl Unitary Trick within E8(8) from SO(16) to SO(8,8).

When Inflation ended 8-dim Octonionic Spacetime was broken into (4+4)-dim Unitary Quaternionic M4 x CP2 Kaluza-Klein Spacetime with SO*(16) symmetry of EIX E8(-24).
That transition was a Weyl Unitary Trick within E8(8) from SO(8,8) to SO*(16) followed by a shifting of SO*(16) symmetry from E8(8) to E8(-24).

In resulting E8 Physics model, the geometry of E8 defines a realistic Classical Local Lagrangian. Since E8 is embedded in the Real Clifford Algebra Cl(16) = Cl(8)xCl(8), 8-Periodiciity allows construction of a generalized Hyperfinite II1 von Neumann factor by taking the completion of the union of all tensor products of Cl(16) which is an Algebraic Quantum Field Theory (AQFT) that is naturally compatible with the realistic E8 Lagrangian. Therefore E8 Physics, which allows calculation of Force Strength and Particle Mass ratios, etc, using the basic ideas of Armand Wyler, fulfills the prediction of Robert Gilmore in his 1974 “long and difficult” book:

> ? (1970–). It now seems possible that Lie group theory, together with differential geometry, harmonic analysis, and some devious arguments, might be able to predict some of Nature’s dimensionless numbers (α, m_p/m_e, m_μ/m_e, G^2/hc, …). In retrospect, it seems clear that the application of group theory to physical problems represents the dividing line between kinematics and dynamics. The group theory gives the overall structure of the spectrum; the dynamics serves to define only the scale. We are looking forward to the day when Lie groups can be pushed to give also the dynamics, or scale.