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E_6, Strings, Branes, and the Standard Model

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Abstract

E_6, an exceptional Lie algebra, contains generators with spinor fermionic characteristics, providing a way to include fermions in string theory without 1-1 boson-fermion supersymmetry. E_6 graded structure $E_6 = g(-2) + g(-1) + g(0) + g(1) + g(2)$ can be used to construct a physically realistic representation of the 26-dimensions of string theory in which strings are interpreted as world-lines in a Many-Worlds quantum picture. This paper describes the construction step-by-step. The resulting physical interpretation is consistent with the physics model described in physics/0207095 [8] .

Contents

1 String Theory, Exceptional Lie Algebra E_6 and Fermions

As usually formulated string theory works in 26 dimensions, but deals only with bosons and so is called bosonic string theory.

Superstring theory as usually formulated introduces fermions through a 1-1 supersymmetry between fermions and bosons, resulting in a reduction of spacetime dimension from 26 to 10.

Pierre Ramond says in his paper Algebraic Dreams at hep-th/0112261 [5] :

> ... M-theory and Superstring theories ... are the only examples of theories where ... union ... [of]... space-time symmetries and internal symmetries appears possible ...

The purpose of this paper is to construct a counterexample to that statement by using the structure of E_6 to build a string theory without 1-1 supersymmetry that nevertheless describes both Gravity and the Standard Model, thus converting Pierre Ramond's Algebraic Dream into an Algebraic Reality.

1.1 26-dim Traceless Jordan J3(O)o and Exceptional Lie algebra E_6

In the same paper, Algebraic Dreams at hep-th/0112261 [5], Pierre Ramond describes the E_6 structure that is used herein to construct a physically realistic string theory that, without 1-1 supersymmetry, describes Gravity and the Standard Model. Pierre Ramond says:

> ... Nature shows that space-time symmetries with dynamics associated with gravity, and internal symmetries with their dynamics described by Yang- Mills theories, can coexist peacefully. How does She do it? ...
>
> Nature relishes unique mathematical structures. ...

> The Exceptional Algebras are most unique and beautiful among Lie Algebras, and no one should be surprised if Nature uses them. ...
>
> The use of exceptional groups to describe space-time symmetries has not been as fruitful [as the use of classical groups] ... One obstacle has been that exceptional algebras relate tensor and spinor representations of their orthogonal subgroups, while Spin-Statistics requires them to be treated differently. ...
>
> The traceless Jordan matrices [J3(O)o] ... (3×3) traceless octonionic hermitian matrices, each labelled by 26 real parameters ... span the 26 representation of [the exceptional Lie algebra] F_4 . One can supplement the F_4 transformations by an additional 26 parameters ... leading to a group with 78 parameters. These extra transformations are non-compact, and close on the F_4 transformations, leading to the exceptional group $E_{6(-26)}$. The subscript in parenthesis denotes the number of non-compct minus the number of compact generators. ...

The construction of this paper uses the exceptional Lie algebra $E_{6(-26)}$, which I denote by E_6, to introduce fermions into string theory in an unconventional way, based on the exceptional E_6 relations between bosonic vectors/bivectors and fermionic spinors, in which 16 of the 26 dimensions are seen as orbifolds whose $8 + 8$ singularities represent first-generation fermion particles and antiparticles.

The exceptional E_6 relations between bosonic vectors/bivectors and fermionic spinors do not establish a 1-to-1 fermion-boson supersymmetry, but do establish an 8-to-28 supersymmetry between first generation fermion particle-antiparticle types and bivector gauge bosons in an 8-dimensional spacetime that, at the low energies of our Earthly human experiments, reduces to a 4-dimensional Internal Symmetry Space and a 4-dimensional Physical Spacetime. Details can be found in physics/0207095 [8] . I call the supersymmetry due to E_6 structure a subtle supersymmetry, to distinguish it from the simple 1-1 supersymmetry that is now widely used.

This structure allows string theory to be physically interpreted as a theory of interaction among world-lines in the Many-Worlds.

1.2 E_6 as a 5-graded Lie algebra

According to Soji Kaneyuki, in Graded Lie Algebras, Related Geometric Structures, and Pseudo-hermitian Symmetric Spaces, which is Part II of the book Analysis and Geometry on Complex Homogeneous Domains, by Jacques Faraut, Soji Kaneyuki, Adam Koranyi, Qi-keng Lu, and Guy Roos (Birkhauser 2000) [2] ,

E_6 can be seen as a Graded Lie Algebra with 5 grades:

$$g = E_6 = g(-2) + g(-1) + g(0) + g(1) + g(2)$$

$g(0) = so(8) + R + R$
$dim_R g(-1) = dim_R g(1) = 16 = 8 + 8$
$dim_R g(-2) = dim_R g(2) = 8$

$g(0) = so(8)+$	$28\ gauge\ bosons$
$+R + R$ $dim_R g(-1) = 16 = 8 + 8$ $dim_R g(-2) = 8$	$26 - dim\ string$ $spacetime$ $with\ J3(O)o\ structure$
$dim_R g(1) = 16 = 8 + 8$ $dim_R g(2) = 8$	$Complexification$ $of\ J3(O)o\ structure$

The E_6 GLA has Even Subalgebra g_E (Bosonic) and Odd Part g_O (Fermionic):

$BOSONIC$	$FERMI0NIC$
$g_E = g(-2) + g(0) + g(2)$	$g_O = g(-1) + g(1)$

1.3 E_6 and 28 gauge bosons, 8-dim spacetime and 8 sets of fermion particles and antiparticles

The graded structure of E_6 describes 28 gauge bosons acting in an 8-dimensional spacetime containing 8 types of fermion particles and 8 corresponding antiparticles. The spacetime and fermion representation space are complex manifolds, but their Shilov boundaries are relevant to physical phenomena as described in physics/0207095 [8] .

BOSONIC structure	*Physical Interpretation*
$g(0) = so(8)$	28 *gauge bosons*
$+R + R$	$1 + 1 -$ *dim Many* $-$ *Worlds degrees of freedom*
$dimRg(-2) = 8$	$8 -$ *dim spacetime*
$dimRg(2) = 8$	*Complexification of* 8 *dim of spacetime*
FERMIONIC structure	
$dimRg(-1) = 16 = 8 + 8 =$ $= 8$ $+$ 8	 $8 -$ *dim fermion Orbifold* $8 -$ *dim antifermion Orbifold*
$dimRg(1) = 16 = 8 + 8 =$ $= 8$ $+$ 8	*Complexification of* $8 -$ *dim fermion Orbifold* *and* $8 -$ *dim antifermion Orbifold*

1.4 4-dim Internal Symmetry Space and 4-dim Physical Spacetime

Dimensional Reduction of 8-dim spacetime to 4-dim Internal Symmetry Space plus 4-dim Physical Spacetime by picking a Quaternionic substructure of Octonionic structure produces 3 generations of fermion particles and antiparticles. Details of the process are in physics/0207095 [8] .

$BOSONIC\ structure$	$Physical\ Interpretation$
$g(0) = so(8)$	$16 - dim\ Conformal\ U(2,2)$ $+$ $12 - dim\ SU(3)xSU(2)xU(1)$
$+R + R$	$1 + 1 - dim\ Many - Worlds\ degrees\ of\ freedom$
$dimRg(-2) = 8 =$ $= 4$ $+$ 4	$4 - dim\ Physical\ Spacetime$ $4 - dim\ Internal\ Symmetry\ Space$
$dimRg(2) = 8 =$ $= 4$ $+$ 4	$Complexification$ $of\ 4 - dim\ Physical\ Spacetime$ and $4 - dim\ Internal\ Symmetry\ Space$
$FERMIONIC\ structure$	$3 - generation\ structure\ of$
$dimRg(-1) = 16 = 8 + 8 =$ $= 8$ $+$ 8	$8 - dim\ fermion\ Orbifold$ $8 - dim\ antifermion\ Orbifold$
$dimRg(1) = 16 = 8 + 8 =$ $= 8$ $+$ 8	$Complexification\ of$ $8 - dim\ fermion\ Orbifold$ and $8 - dim\ antifermion\ Orbifold$

2 Strings, Branes, and the Standard Model

The E_6 Structure described in Section 1 allows construction of a realistic string theory.

The construction was motivated by a March 2004 sci.physics.research thread Re: photons from strings? [6]
in which:

John Baez asked [6]

> ... has anyone figured out a way to ... start with string theory ... to get just photons on Minkowski spacetime ... ?

Lubos Motl noted [6]

> ... string theory always contains gravity ... Gravity is always contained as a vibration of a closed string, and closed strings can always be created from open strings ...

Urs Schreiber said [6]

> ... the low energy effective worldsheet theory of a single flat D3 brane of the bosonic string is, to lowest nontrivial order, just $U(1)$ gauge theory in 4D ...

Aaron Bergman noted [6]

> ... there are a bunch of scalars describing the transverse fluctuations of the brane ...

Urs Schreiber said [6]

> . . . I guess that's why you have to put the brane at the singularity of an orbifold if you want to get rid of the scalars . . . if the number of dimensions is not an issue the simplest thing probably would be to consider the single space-filling D25 brane of the bosonic string. This one does not have any transverse fluctuations and there is indeed only the $U(1)$ gauge field . . .

Aaron Bergman replied [6]

> . . . Unfortunately, there's a tadpole in that configuration. You need 8192 D25 branes to cancel it. . . .

Lubos Motl pointed out the existence of brane structures other than massless vectors, saying [6]

> . . . A D-brane contains other massless states, e.g. the transverse scalars (and their fermionic superpartners). It also contains an infinite tower of excited massive states. Finally, a D-brane in the full string theory is coupled to the bulk which inevitably contains gravity as well as other fields and particles. . . . N coincident D-branes carry a $U(N)$ gauge symmetry (and contain the appropriate gauge N^2 bosons, as you explained). Moreover, if this stack of N D-branes approaches an orientifold, they meet their mirror images and $U(N)$ is extended to $O(2N)$ or $USp(2N)$. The brane intersections also carry new types of matter - made of the open strings stretched from one type of brane to the other - but these new fields are *not* gauge fields, and they don't lead to new gauge symmetries. For example, there are scalars whose condensation is able to join two intersecting D2-branes into a smooth, connected, hyperbolically shaped objects (D2-branes). . . . the number of D-branes can be determined or bounded by anomaly cancellation and similar requirements. For example, the spacetime filling D9-branes in type I theory must generate the $SO(32)$ gauge group, otherwise the theory is anomalous. (There are other arguments for this choice of 16+16 branes, too.) . . .

Motivated by the above remarks, this paper constructs a specific example of a String Theory with E_6 structure containing gravity and the $U(1) \times SU(2) \times SU(3)$ Standard Model.

As to how the simple model is affected by some of the complications mentioned by Lubos Motl:

- Transverse scalars are taken care of by Orbifolding as suggested by Urs Schreiber.
- Fermionic superpartners are taken care of by not using simple 1-1 fermion-boson supersymmetry.
- The infinite tower of excited massive states is related to Regge trajectories which in turn are related to interactions among strings considered as world-lines in the Many-Worlds.
- Bulk gravity is included.
- There are no orientifolds.
- Open strings from one brane to another, as vacuum loops, look like exchange of closed loops and are related to gravity among branes and the Bohm-type quantum potential.
- Scalar condensates are related to Dilatons which in turn are related to interactions among strings considered as world-lines in the Many-Worlds.
- I have not fully investigated all potential anomaly problems.

Further, string theory Tachyons are related to interactions among strings considered as world-lines in the Many-Worlds.

In short, the complications, with the possible exception of some anomalies, are either taken care of in the construction of the model or are useful in describing the Bohm-type quantum potential interactions among strings considered as world-lines in the Many-Worlds.

Here is some further background, from Joseph Polchinski's book String Theory vol. 1 (Cambridge 1998), in Chapter 8 and the Glossary [4] :

> ... a ... D-brane ... [is] ... a dynamical object ... a flat hyperplane ... [for which] ... a certain open string state corresponds to fluctuation of its shape ...
>
> A D25-brane fills space, so the string endpoint can be anywhere ...
>
> When no D-branes coincide there is just one massless vector on each, giving the gauge group $U(1)^n$ in all.
>
> If r D-branes coincide, there are new massless states because string that are stretched between these branes can have vanishing length ... Thus, there are r^2 vectors, forming the adjoint of a U(r) gauge group. ... there will also be r^2 massless scalars from the components normal to the D-brane. ...
>
> The massless fields on the world-volume of a Dp-brane are a U(1) vector plus 25 - p world-brane scalars describing the fluctuations. ... The fields on the brane are the embedding $X^{u(x)}$ and the gauge field $A_{a(x)}$ ...
>
> For n separated D-branes, the action is n copies of the action for a single D-brane. ... when the D-branes are coincident there are n^2 rather than n massless vectors and scalars on the brane ...
>
> The fields $X^{u(x)}$ and $A_{a(x)}$ will now be nxn matrices ...
>
> the gauge field ... becomes a non-Abelian U(n) gauge field ...
>
> the collectives coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to $n \times n$ matrices. This 'non-commutative geometry' ... [may be] ... an important hint about the nature of spacetime. ...
>
> [an] ... orbifold ...
>
> (noun) ... [is] ... a coset space M/H, where H is a group of discrete symmetries of a manifold M. The coset is singular at the fixed points of H ...

(verb) . . . [is] . . . to produce such a . . . string theory by gauging H . . .

To determine the actual value of the D-brane tension . . . Consider two parallel Dp-branes . . . [They] . . . can feel each other's presence by exchanging closed strings . . . [which is equivalent to] . . . a vacuum loop of an open string with one end on each D-brane . . . The . . . analogous . . . field theory graph . . . is the exchange of a single graviton or dilaton between the D-branes . . .

Here, step-by-step, is the String/Brane construction:

2.1 Step 1 - 26-dim J3(O)o

Consider the 26 Dimensions of String Theory as the 26-dimensional traceless part J3(O)o of the 27-dimensional Jordan algebra J3(O) of 3x3 Hermitian Octonion matrices.

$$\begin{pmatrix} a & \mathbf{O}_+ & \mathbf{O}_v \\ \mathbf{O}_+^\dagger & b & \mathbf{O}_- \\ \mathbf{O}_v^\dagger & \mathbf{O}_-^\dagger & -a-b \end{pmatrix} \tag{1}$$

where Ov, O+, and O- are in Octonion space with basis 1,i,j,k,E,I,J,K and a and b are real numbers with basis 1 of the 27-dimensional Jordan algebra J3(O) of 3x3 Hermitian Octonion matrices.

2.2 Step 2 - D3 of Quaternionic substructure of Octonionic space

Take Urs Schreiber's D3 brane to correspond to the Imaginary Quaternionic associative subspace spanned by i,j,k in the 8-dimenisonal Octonionic Ov space.

2.3 Step 3 - Compactify 4-dim Internal Symmetry Space and make D7

Compactify the 4-dimensional co-associative subspace spanned by E,I,J,K in the Octonionic Ov space as a $CP^2 = SU(3)/U(2)$, with its 4 world-brane scalars corresponding to the 4 covariant components of a Higgs scalar.

Add this subspace to D3, to get D7.

2.4 Step 4 - Orbifold to discretize time and make D8

Orbifold the 1-dimensional Real subspace spanned by 1 in the Octonionic Ov space by the discrete multiplicative group Z2 = -1,+1, with its fixed points

-1,+1 corresponding to past and future time. This discretizes time steps and gets rid of the world-brane scalar corresponding to the subspace spanned by 1 in Ov. It also gives our brane a 2-level timelike structure, so that its past can connect to the future of a preceding brane and its future can connect to the past of a succeeding brane.

Add this subspace to D7, to get D8.

D8, our basic Brane, looks like two layers (past and future) of D7s.

Beyond D8 our String Theory has 26 - 8 = 18 dimensions, of which 25 - 8 have corresponding world-brane scalars:

- 8 world-brane scalars for Octonionic O+ space;
- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

2.5 Step 5 - Orbifold to discretize fermion particles

To use Urs Schreiber's idea
to get rid of the world-brane scalars corresponding to the Octonionic O+ space, orbifold it by the 16-element discrete multiplicative group
Oct16 = +/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K to
reduce O+ to 16 singular points -1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K.

- Let the 8 O+ singular points -1,-i,-j,-k,-E,-I,-J,-K correspond to the fundamental fermion particles
  neutrino, red up quark, green up quark, blue up quark,
  electron, red down quark, green down quark, blue down quark
  located on the past D7 layer of D8.
- Let the 8 O+ singular points +1,+i,+j,+k,+E,+I,+J,+K correspond to the fundamental fermion particles
  neutrino, red up quark, green up quark, blue up quark,
  electron, red down quark, green down quark, blue down quark
  located on the future D7 layer of D8.

This gets rid of the 8 world-brane scalars corresponding to O+, and leaves:

- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

2.6 Step 6 - Orbifold to discretize fermion anti-particles

To use Urs Schreiber's idea to get rid of the world-brane scalars corresponding to the Octonionic O- space, orbifold it by the 16-element discrete multiplicative group
Oct16 = +/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K to
reduce O- to 16 singular points -1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K.

- Let the 8 O- singular points -1,-i,-j,-k,-E,-I,-J,-K correspond to the fundamental fermion anti-particles
  anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark,
  positron, red down anti-quark, green down anti-quark, blue down anti-quark
  located on the past D7 layer of D8.
- Let the 8 O- singular points +1,+i,+j,+k,+E,+I,+J,+K correspond to the fundamental fermion anti-particles
  anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark,
  positron, red down anti-quark, green down anti-quark, blue down anti-quark
  located on the future D7 layer of D8.

This gets rid of the 8 world-brane scalars corresponding to O-, and leaves:

- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

2.7 Step 7 - Strings as world-lines

Let the 1 world-brane scalar for real a space correspond to a Bohm-type Quantum Potential acting on strings in the stack of D8 branes.

Interpret strings as world-lines in the Many-Worlds, short strings representing virtual particles and loops.

2.8 Step 8 - Fundamental lattice structure

Fundamentally, physics is described on HyperDiamond Lattice structures.

There are 7 independent E_8 lattices, each corresponding to one of the 7 imaginary octionions and each with 240 first-shell vertices
and related to both the D8 ajoint and half-spinor parts of E8.

They can be described as $iE_8, jE_8, kE_8, EE_8, IE_8, JE_8, KE_8$.

Further, an 8th 8-dim lattice 1E8 with 240 first-shell vertices related
to the D8 adjoint part of E8 is related to the 7 octonion imaginaries.

Give each D8 brane structure based on Planck-scale E_8 lattices so that each D8 brane is a superposition/intersection/coincidence of the eight E_8 lattices. (this section revised February 2013 - see viXra 1301.0150v2)

2.9 Step 9 - r coinciding D-branes give a $U(r)$ gauge group

Since J. Polchinski says [4]

> ...If r D-branes coincide ...there are r^2 vectors, forming the adjoint of a U(r) gauge group. ...

make the following assignments:

- a gauge boson emanating from D8 only from its $1E_8$ lattice is a $U(1)$ photon;
- a gauge boson emanating from D8 only from its $1E_8, EE_8$ lattices is a $U(2)$ weak boson;
- a gauge boson emanating from D8 only from its IE_8, JE_8, KE_8 lattices is a $U(3)$ gluon.

Note that I do not consider it problematic to have $U(2)$ and $U(3)$ instead of $SU(2)$ and $SU(3)$ for the weak and color forces, respectively. For my reasoning, involving the global Standard Model group structure and the root vector structures of the Standard Model groups, see physics/0207095 [8] .

2.10 Step 10 - Components normal to the D-brane

Since Joseph Polchinski says [4]

> ...there will also be r^2 massless scalars from the components normal to the D-brane. ...the collectives coordinates ... X^u ...for the embedding of n D-branes in spacetime are now enlarged to $n \times n$ matrices. This 'noncommutative geometry' ...[may be] ...an important hint about the nature of spacetime. ...

make the following assignment:

The 8×8 matrices for the collective coordinates linking a D8 brane to the next D8 brane in the stack are needed to connect the eight E_8 lattices of the D8 brane to the eight E_8 lattices of the next D8 brane in the stack.

We have now accounted for all the scalars, and, since, as Lubos Motl [6] ,

> ...string theory always contains gravity ...

we have here at Step 10 a specific example of a String Theory containing gravity and the $U(1) \times SU(2) \times SU(3)$ Standard Model.

2.11 Step 11 - Conformal gravity of I. E. Segal

We can go a bit further by noting that we have not described gauge bosons emanating from D8 from its $1E_8, iE_8, jE_8, kE_8$ lattices.

Therefore, make the following assignment:

- a gauge boson emanating from D8 only from its $1E_8, iE_8, jE_8, kE_8$ lattices is a $U(2,2)$ conformal gauge boson.

We have here at Step 11 a String Theory containing the Standard Model plus two forms of gravity:

- closed-string gravity and
- conformal $U(2,2) = Spin(2,4)xU(1)$ gravity plus conformal structures, based on a generalized MacDowell-Mansouri mechanism.

I conjecture that those two forms of gravity are not only consistent, but that the structures of each will shed light on the structures of the other, and that the conformal structures are related to the conformal gravity ideas of I. E. Segal, set out in his book Mathematical Cosmology and Extragalactic Astronomy (Academic Press 1976) and in some other references given at web page http://www.innerx.net/personal/tsmith/SegalConf.html and related pages [7].

2.12 Step 12 - Exceptional Lie algebras E_6, E_7 and E_8

Going a bit further leads to consideration of the exceptional E-series of Lie algebras, as follows:

a gauge boson emanating from D8 only from its $1E_8, iE_8, jE_8, kE_8, EE_8$ lattices is a $U(5)$ gauge boson related to $Spin(10)$ and Complex E_6.

a gauge boson emanating from D8 only from its $iE_8, jE_8, kE_8, EE_8, IE_8, JE_8, KE_8$ lattices is a $U(6)$ gauge boson related to $Spin(12)$ and Quaternionic E_7.

a gauge boson emanating from D8 only from its $1E_8, iE_8, jE_8, kE_8, EE_8, IE_8, JE_8$ lattices is a $U(7)$ gauge boson related to $Spin(14)$ and possibly to Sextonionic $E_{7\frac{1}{2}}$.

a gauge boson emanating from D8 only from its $1E_8, iE_8, jE_8, kE_8, EE_8, IE_8, JE_8, KE_8$ lattices is a $U(8)$ gauge boson related to $Spin(16)$ and $Octonionic E_8$.

These correspondences are based on the natural inclusion of $U(N)$ in $Spin(2N)$ and on Magic Square constructions of the E series of Lie algebras, roughly described as follows:

- 78-dim E_6 = 45-dim Adjoint of $Spin(10)$ + 32-dim Spinor of $Spin(10)$ + Imaginary of $\mathbf{C}$;

- 133-dim E_7 = 66-dim Adjoint of $Spin(12)$ + 64-dim Spinor of $Spin(12)$ + Imaginaries of **Q**;
- 248-dim E_8 = 120-dim Adjoint of $Spin(16)$ + 128-dim half-Spinor of $Spin(16)$.

Physically,

- E_6 corresponds to 26-dim String Theory,
  related to traceless J3(O)o and the symmetric space E_6/F_4.
- E_7 corresponds to 27-dim M-Theory,
  related to the Jordan algebra J3(O) and the symmetric space $E_7/E_6 \times U(1)$.
- E_8 corresponds to 28-dim F-Theory, related to
  the Jordan algebra J4(Q) and the symmetric space $E_8/E_7 \times SU(2)$.

3 Notes

3.1 Note on Sextonions

I am not yet clear about how the Sextonionic $E_{7\frac{1}{2}}$ works. It was only recently developed by J. M. Landsberg and Laurent Manivel in their paper "The sextonions and $E_{7\frac{1}{2}}$" at math.RT/0402157 [3]. Of course, the Sextonion algebra is not a real division algebra, but it does have interesting structure. In their paper, Landsberg and Manivel say:

> ...We fill in the "hole" in the exceptional series of Lie algebras that was observed by Cvitanovic, Deligne, Cohen and deMan. More precisely, we show that the intermediate Lie algebra between E_7 and E_8 satisfies some of the decomposition and dimension formulas of the exceptional simple Lie algebras. A key role is played by the sextonions, a six dimensional algebra between the quaternions and octonions. Using the sextonions, we show simliar

> results hold for the rows of an expanded Freudenthal magic chart. We also obtain new interpretations of the adjoint variety of the exceptional group G_2. ...the orthogonal space to a null-plane U, being equal to the kernel of a rank-two derivation, is a six-dimensional subalgebra of O. ...The decomposition ...into the direct sum of two null-planes, is unique. ...[this] ...provides an interesting way to parametrize the set of quaternionic subalgebras of O. ...

Some possibly related facts of which I am aware include:

- The set of Quaternionic subalgebras of Octonions = $SU(3) = G_2/Spin(4)$.
- $G_2/SU(3) = S^6$ is almost complex but not complex and is not Kaehler. Its almost complex structure is not integrable.
  See chapter V of Curvature and Homology, rev. ed., by Samuel I. Goldberg (Dover 1998) [1] .
- It may be that the sextonions and S^6 are related to $Spin(4)$ as the 6-dim conformal vector space of $SU(2,2) = Spin(2,4)$ is related to 4-dim Minkowski space.

3.2 Note on the Monster

The 26 dimensions of String Theory might be related to the 26 Sporadic Finite Simple Groups, the largest of which, the Monster, has about 8×10^{53} elements. If you use positronium (electron-positron bound state of the two lowest-nonzero-mass Dirac fermions) as a unit of mass Mep = 1 MeV, then it is interesting that the product of the squares of the Planck mass Mpl = 1.2×10^{22} MeV and W-boson mass Mw = 80,000 MeV gives $((M_{pl}/M_{ep})(M_w/M_{ep}))^2 = 9 \times 10^{53}$ which is roughly the Monster order. Maybe the Monster shows how, in the world of particle physics, "big" things like Planck mass and W-bosons are related to "little" (but not zero-mass) things like electrons and positrons, thus giving you some persepective on the world of fundamental particles.

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physics/0207095

Note added 18 July 2005:

Lubos Motl, in his blog entry Tachyons and the Big Bang at http://motls.blogspot.com/ dated 13 July 2005, said:
"... closed string tachyons ... signal an instabilityof the whole spacetime ...
... closed string tachyons ... can be localized ifthey appear in a twisted sector of an orbifold ...
the twisted closed strings describe fields thatare localized at the origin ...
The tachyons condense near the tip which smears out the tip of the cone which makes the tip nice and round. ...".

Closed string tachyons localized at an orbifold may be physically equivalent to what Schroer describes in http://xxx.lanl.gov/abs/hep-th/9908021 as "... any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles, unless the system is free ..."
and to
Dirac's 1938 Dirac-Lorentz equation model of the electron as described in pages 194-195 of
Dirac: A Scientific Biography,by Helge Kragh (Cambridge 1990):
"... Dirac explained that the strange behavior of electrons in this theory could be understood if the electron was thought of as an extended particlewith a nonlocal interior.
He suggested that the point electron, embedded in its own radiation field, be interpreted as a sphere of radius a, where a is the distance within which an incoming pulse must arrive before the electron accelerates appreciably.
With this interpretation he showed that it was possible for a signal to be propagated faster than light through the interior of the electron. ... In spite of the appearance of superluminal velocities, Dirac's theorywas Lorentz-invariant. ...".

In short, if orbifolds are identified with fermion particles,
then their localized tachyons can be physically interpreted as describing the virtual particle-antiparticle clouds that dress the fermion particles.